Multiple Model Adaptive Control Design for a MIMO Chemical Reactor

R. Gundala  
Department of Chemical Engineering  
University of South Carolina  
Columbia, SC 29208

K. A. Hoo*  
Department of Chemical Engineering  
Texas Tech University  
Lubbock, TX 79409

M.J. Piovoso  
DuPont Central Research & Development  
DuPont Chemical Company  
Wilmington, DE 19880-0101

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*Author to whom all correspondence should be addressed. Ph:(806)742-4079, Fax: (806) 742-3552, Email: khoo@coe.ttu.edu
Abstract

Multiple adaptive and nonadaptive models are used to represent the behavior of processes that are known to transition to unknown regimes in the operating space. The adaptive models investigated are of two types, free-running and re-initializable, that differ in the initialization of their parameters. Using these models and their companion controllers in a model reference adaptive structure, it is shown that this mixed model set can provide satisfactory control of a nonlinear, interactive, multiple-input multiple-output chemical reactor with active constraints.
1 Introduction

Many challenging industrial process control problems have been published by chemical companies genuinely interested in knowing how advanced control theories can be implemented in real situations. Among them is the Tennessee Eastman challenge problem that naturally embodies nonlinearities, product transitions, constraints, and time delays. Excellent studies have appeared since the time of its publication [1]. These include, the multiple proportional-integral (PI) control strategy developed by McAvoy and Ye [2]; a decentralized model predictive controller strategy by Ricker [3]; a nonlinear model predictive controller strategy and a steady state model with state estimation by Ricker and Lee [4, 5]; and an inferential controller design by Ye et al. [6].

The present work deals with a simplified version of the Tennessee Eastman (TE) process that was proposed by Ricker [7]. This process consists of a two-phase reactor so as to capture the essential nonlinear features of the TE problem. The TE process and the nonlinear two-phase reactor are processes that naturally transition to different and unknown regions in the operating space either by design (set point changes) or because of unplanned events (disturbances). The dynamic behavior of the system changes with changing operating conditions. The control problem is further complicated by a hard constraint on the reactor pressure. If violated, the reactor is shut down for safety reasons.

One means of addressing this problem is to employ an adaptive control strategy. The objective of any adaptive system is to provide an accurate representation of the process at all times. An adaptive system has maximum application when the plant undergoes transitions or exhibits nonlinear behavior and when the structure of the plant itself is not known. Gain scheduling is one form of adaptive control but it requires knowledge about all the transitions to be effective [8]. Another alternative is to adapt the controllers parameters or when a model is available, use the model identification errors to tune the controller’s parameters [8, 9, 10]. Consequently, tuning of the controller
is indirect and necessarily requires an accurate model of the process for satisfactory performance. The concept of using multiple models especially when the operating conditions are non-stationary is another option. In most cases, the notions of switching and an update law to change the model and/or controller parameters are also involved. For instance, multiple models were used to improve the accuracy of state estimation by Magill [11] and Middelton et al. used multiple models with various switching concepts to achieve stability with minimal prior information [12]. Morse applied multiple models and a state-shared controller for robust set point control [13]; and Kosanovich et al. demonstrated this approach on a nonlinear, open-loop, unstable chemical reactor that transitioned among three steady-state regions in the operating space [14]. Sun et al. developed a supervisory transition control strategy that consisted of multiple nonadaptive models and controllers with stable switching to control a class of SISO nonlinear processes with and without time delays [15, 16]; and more recently, Narendra and Balakrishnan and Balakrishnan proposed multiple and more than one type of adaptive model and controllers to control SISO linear systems [17, 18, 19].

The present work uses combinations of both multiple nonadaptive and adaptive models and their controllers, in a model reference structure, to control the actual nonlinear process. If the adaptive models give better performance than the nonadaptive ones then their controllers will be used to control the nonlinear process.

The organization of this paper is as follows. First, the multiple model and controller approach is developed. Second, theorems about stability of the nonadaptive and adaptive models and stable switching are provided but without proofs, for completeness. Next, the multiple model reference adaptive structure (MMRAS) is applied to a MIMO nonlinear chemical reactor in the presence of unmeasured disturbances, parametric drifts, and production rate changes and compared to a multiple PI (MPI) controller strategy. The final section discusses the results obtained and the
future research issues.

2 Multiple Adaptive Models and Controllers

The multiple models and controllers, as proposed by Narendra and Balakrishnan [17, 18] and Balakrishnan [19], may consist of nonadaptive or adaptive or a combination of both types of models and controllers. To paraphrase Narendra, “the rationale for using multiple models is clear, that is, to ensure that there is at least one model with parameters sufficiently close to those of the unknown plant.” Furthermore, by having a combination of nonadaptive and adaptive models that cover the operating space of the system, even highly nonlinear systems, such as the one to be studied, can be well controlled and transitioned from one part of the operating space to another.

The adaptive models may be further categorized according to where they begin their parameter adaptation. The conventional adaptive model begins its adaptation from its initial parameter set, a free-running adaptive model will start from its current parameter values, and a re-initializable adaptive model will assume the parameter set of the nonadaptive model that gives the smallest identification error. Whatever choice is made accuracy, speed, and stability of the closed-loop performance should be satisfactory.

With multiple nonadaptive models, uniformly distributed in the operating space, it is possible to achieve a timely response as the computational burden associated with convergence and parameter updates is avoided [18]. With only adaptive models accuracy can be achieved, however convergence may be slow. With both model types present, speed and accuracy are possible.

For different combinations of model types, Narendra and Balakrishnan and Balakrishnan provide the necessary theorems that assure stability [18, 19]. The interested reader can find the proofs in the manuscripts cited.
2.1 Model Structures

The model reference structure approach requires a model whose output provides a reference trajectory for the output of the process to follow [8, 9, 10]. Without loss of generality, a SISO linear system is used to describe the multiple model reference adaptive approach. The development follows that of Narendra and Balakrishnan [17, 18] and are presented here for review and completeness.

The state space representation of the linear SISO process to be controlled is given by

\[
\begin{align*}
\frac{dx}{dt} &= A(p)x(t) + B(p, \theta)u(t) \\
y(t) &= h(p)x(t)
\end{align*}
\]

where \(x \in \mathbb{R}^n\) are the states, \(u \in \mathbb{R}\) is the input, \(y \in \mathbb{R}\) is the output, \(\{A, B, h\}\) are vector valued functions, and the unknown plant parameters belong to a compact set \(S\). The elements of \(p\) are either unknown and constant or vary with time while the elements of \(\theta\) represent parameters that are under the control of the designer. The parameters, \(\theta\), are used for control purposes to define the process input \(u\).

Suitably parameterized models, \(M_j(p_j), j = 1, \ldots, N; p \in S\) of the system given in Equation (1), can be developed, operating in parallel, but with different initial estimates of the process parameters. Some of these models may have nonadaptive parameter values or their parameters are allowed to adapt starting from some initial value. The input to all the models is \(u\) and their outputs are given by \(y_j, j = 1, \ldots, N\).

For each nonadaptive model, a controller, \(C_j(\theta_j), j = 1, \ldots, N\) is designed with parameters \(\theta\). It is assumed that if the output of model \(M_j(p_j)\) gives the smallest identification error, then its companion controller will be the best to use for controlling the process. Moreover, it is also assumed that each one of the \(N\) controllers if used alone results in local stability at their operating condition. If the models are adaptive, then the model and control parameters, \(\{p_j, \theta_j\}\), are tuned simultaneously.
The system in Equation (1) can be represented by a ratio of rational polynomials,

\[ W_p(s) = \frac{k_p Z_p(s)}{D_p(s)} \]  \hspace{1cm} (2)

where \( k_p \) is the gain of the system and the coefficients of \( Z_p \) and \( D_p \) constitute the unknown plant parameter vector \( p \in S \), a compact set in \( \mathbb{R}^{2n} \) where \( n \) is the order of the linear system. \( D_p(s) \) and \( Z_p(s) \) are monic, coprime polynomials with degrees \( n \) and \( m(\leq n) \), respectively and \( Z_p(s) \) is a Hurwitz polynomial [20].

Similarly, define the reference model to be a linear time-invariant (LTI) process given by

\[ W_r(s) = \frac{k_r Z_r(s)}{D_r(s)} \]  \hspace{1cm} (3)

where \( k_r \) is the gain, and both \( Z_r(s) \) and \( D_r(s) \) are monic, coprime Hurwitz polynomials. The reference models define the desired behavior of the closed-loop system. Choose the degree of \( D_r(s) \) to be \( n \) and that of \( Z_r(s) \) to be \( m \). Let the input signal to the reference model, \( r(t) \), belong to the class of bounded piecewise differentiable inputs and denote the output of the reference model to be \( y_r(t) \).

Within the model reference structure, boundedness of the signals in the overall system and asymptotic convergence of the control error defined as

\[ e_c(t) \equiv y_p(t) - y_r(t) \]  \hspace{1cm} (4)

are achieved using a differentiator-free control input \( u(t) \) [21, 9].

A new LTI system is defined with states, \( \omega_1(t), \omega_2(t) \in \mathbb{R}^{n-1} \), and driven by external signals, \( u(t) \) and \( y_p(t) \),

\[
\begin{bmatrix}
\frac{d\omega_1}{dt} \\
\frac{d\omega_2}{dt}
\end{bmatrix} = \Lambda \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \ell \begin{bmatrix} u \\ y_p \end{bmatrix} \]
\]  \hspace{1cm} (5)
that ties together the plant and the model reference responses where \((\Lambda, \ell)\) are in controllable form [20, 22]. The coefficients of the characteristic equation are chosen such that the determinant of this system contains \(Z_r(s)\) as a factor. That is,

\[
det(sI - \Lambda) = \lambda(s)Z_r(s)
\]  

(6)

where \(\lambda(s)\) is a monic, Hurwitz polynomial of degree \(n - m - 1\).

Since the numerator and denominator polynomials of \(W_p\) do not share any common factors, there exists unique polynomials \(\alpha(s)\) and \(\beta(s)\) \(\in \mathbb{R}^{n-1}\), such that the following identity, known as the Bezout Identity or in algebra the Diophantine equation, holds [8]

\[
[W_p(s)\lambda(s)Z_r(s)]^{-1} \alpha(s) + [\lambda(s)Z_r(s)]^{-1} \beta(s) = W_r(s)^{-1}
\]  

(7)

Using the above, the process output can be expressed in terms of the reference model and the polynomials, \(\alpha(s)\) and \(\beta(s)\), if the plant transfer function is known precisely. That is,

\[
y_p(s) = W_r(s)p^*(s)\omega(s)
\]

(8)

where \(p^*(s) = [\beta^*(s) \quad \alpha^*(s)]\) with \(\beta^*_0\) the ratio of the DC gains of the plant and reference model, and \(\alpha^*_0\) a constant.

Similarly, the output of each model, \(y_j(t)\), can be placed in this form,

\[
\hat{y}_j(s) = W_r(s)p^j(s)\omega(s)
\]

where \(y_j(s), \hat{p}_j(s)\) are estimates of \(y_p\) and \(p^*\), respectively. It then follows, that the parameter and identification errors for \(M_j\) and the input error for \(C_j\) can be defined as

\[
\hat{\beta}_j(t) \equiv \hat{p}_j(t) - p^*(t)
\]

(9)

\[
e_j(t) \equiv y_j(t) - y_p(t)
\]

(10)

\[
\hat{u}_j(t) \equiv u_j(t) - u^*(t)
\]

(11)
respectively where $u^*(t)$ is the ideal input to the process that results in the output of the process tracking the reference model output,
\[
\lim_{t \to \infty} y_p(t) \to y_r(t).
\]

2.2 The Controller Design

If the ideal parameter vector, $p^*(t)$ is known, then the ideal input required to cause the plant to track the reference model is computed as follows
\[
\begin{align*}
& \hat{u}^*(s) \equiv \theta^*(s) \omega_r(s) \\
& \omega'_r(s) = [r \quad \omega'_1 \quad y_p \quad \omega_2] \\
& \theta^*(s) = (\beta_0^*)^{-1} [-1 \quad \beta_1^* \quad \alpha_0^* \quad \alpha_2^*] = [k_c^* -\theta_0^* -\theta_1^* -\theta_2^*] \\
\end{align*}
\]
However, when the models are adaptive the parameters, $\hat{p}_j(t)$, must be tuned. The output of each $C_j$ is given by
\[
\begin{align*}
& u_j(t) = \theta_j(t) \omega_r(s) \\
\end{align*}
\]
with a suitable error equation for the controller’s parameters defined as
\[
\tilde{\theta}_j(t) \equiv \theta_j(t) - \theta^*(t) \quad j = 1, \ldots, N.
\]
Substitution of Equations (8), (13), and (14) into (4) gives the control error in terms of input signals $y_p$ and $r$
\[
e_r(s) = W_r(s)\beta_0^* \theta'(s) \omega_r(s).
\]
In the work by Narendra and Balakrishnan theorems are provided that demonstrate that the error between the selected controller input and the ideal one is explicitly related to the errors in the model parameters [18]. These in turn are tied explicitly to the errors in the estimates of the controller parameters when the model is adaptive. Thus, small model parameter errors imply small controller output errors, and small controller parameter errors imply small model parameter errors.
In ref [17], update laws that are a result of using a Lyapunov stability argument, guarantee that as \( t \to \infty \), the closed-loop parametric errors of the adaptive controllers asymptotically approach zero. It then follows that the control and the identification errors also asymptotically approach zero. Furthermore, since all the \( \theta_j(t), j = 1, \ldots, N \) are bounded, the states of the overall system can grow at most exponentially. This assures existence of a unique solution on the interval \( t \in [0, \infty) \).

3 Stability

It is possible that switching between stabilizing controllers need not result in a stable system. This issue was investigated by Narendra [21], Morse [13], and Sun et al. [15]. The theoretical foundation to establish stability in a multiple model reference adaptive structure depends on the choice of the models. The theorems are provided for completeness. Their proofs can be found in [17, 18].

3.1 Nonadaptive Models

In the case of only nonadaptive models, stability is not guaranteed for arbitrary switching. Further assume that \( M_j(p_j) \), activated at time \( t \), gives the smallest identification error or the minimum performance index and that it cannot be deactivated until time \( t + T_{min} \). It is possible that this controller may not give the minimum performance index during the waiting period, \( [t, t + T_{min}] \). Hence, the selection of \( T_{min} \) must be sufficiently small to limit such deviations.

Let

\[
\Phi_j(t) = v_1 e_j^2(t) + v_2 \int_0^t e^{-\lambda(t-\tau)} e_j^2(\tau) d\tau
\]

(16)

define the performance index of any model at any instant. This choice guarantees that both transient errors and long term (steady-state) behavior will be quickly detected without frequent
switching. The switching scheme is then based on monitoring $\Phi_j(t)$, with the requirement that $T_{\text{min}}$ must elapse before the controller corresponding to the minimum $\Phi_j(t)$ is switched into feedback with the process. Stable control of the identified model will lead to stable control of the actual process.

**Theorem 1**: Consider the model and controller development described in §2, and that the $N$ models are all nonadaptive. Assume that Equation (16), with finite constants $v_2, \lambda, T_{\text{min}} > 0$ and $v_1 \geq 0$, is used with any stable switching scheme. Then, for each process with parameter vector $p \in S$, there exists a $T_s > 0$ that depends only on $S$ and a function $\mu_S(p, T_{\text{min}}) \geq 0$, that depends upon $v_1, v_2, \lambda$, and $S$ such that if $T_{\text{min}} \in (0, T_S)$ and there is at least one model $M_k, k = 1, \ldots, N$ with parameter error $\|\hat{p}_k - p\| < \mu_S(\tilde{p}_k, T_{\text{min}})$ then all the signals in the overall system, as well as the performance indices $\Phi_j(t), j = 1, \ldots, N$, are uniformly bounded.

The proof rests on showing that the state of the closed-loop system can grow at most exponentially. By hypothesis, there exists at least one model/controller pair with parameter error $\|\hat{p}_k - p\| < \mu_S(\tilde{p}_k, T_{\text{min}})$ such that by choosing $\mu_S$, the uncertainty radii which assures stability for every process in the set $S$ appropriately, $e_k(t)$ can be made sufficiently small as compared to the state itself.

### 3.2 Nonadaptive Models and One Free-running Adaptive Model

To satisfy stability and tracking requires the existence of at least one nonadaptive model. This necessitates a large number of models which is undesirable from a practical point of view. Since the concept of an adaptive model means that its parameters can be modified to lie near the ideal parameters $p^*$, this concern can be addressed with the inclusion of an adaptive model that operates in parallel with the nonadaptive models. This additional model does not alter the switching scheme.
The type of adaptive model to be considered is a free-running adaptive model that begins to tune its parameters starting from its current state.

Any tuning scheme can be used as long as the following identification conditions are satisfied [18].

(a) The parameters of the adaptive model, \( M_F \), and controller, \( C_F \), are bounded, \( \hat{p}_F(t), \theta_F(t) \in L_\infty \).
(b) The rate of convergence of the parameters is also bounded, \( \dot{\hat{p}}_F(t), \dot{\theta}_F(t) \in L_\infty \cap L_2 \).
(c) The identification error, \( e_F \in L_2 \).
(d) The closed-loop parametric errors converge asymptotically to zero.

The condition that the identification error of the adaptive model is sufficiently small after a finite time is well established [17]. In the case of all nonadaptive models, this was satisfied only by having a large number of models. The following theorem guarantees stability for this model combination.

**Theorem 2**: Consider the model and controller development described in §2, with \( N_1 \) nonadaptive models and \( N_2 \geq 1 \) free-running adaptive models, where the latter are assumed to satisfy the identification conditions stated above. Let the switching scheme, described above, be used with \( \nu_2, \lambda, \) and \( T_{\text{min}} > 0 \). There exists a \( T_S > 0, \) that depends only on the set \( \mathcal{S} \), such that if \( T_{\text{min}} \in (0, T_S) \), then all the signals in the overall system including the performance indices \( \Phi_j(t) \mid j = 1, \ldots, N_1 + N_2 \), are uniformly bounded.

The proof is similar to that used in theorem 1. Narendra and Balakrishnan point out that the above theorem does not mean that the control error asymptotically converges to zero because there are no guarantees that the adaptive model will be selected when there are nonadaptive models that are close to the operating conditions [18]. The choice of the controller may be oscillatory (chattering) under these circumstances.
3.3 Nonadaptive, Free-running and Re-initializable Adaptive Models

One limitation posed by the above combination of models is that large transient errors may occur due to initial parametric errors. This also impacts the time to convergence when the initial errors are large. The addition of a re-initializable model can improve the response because this type of adaptive model is allowed to adapt starting with parameters of the nonadaptive model that has the smallest performance index. This means that once a nonadaptive model, $M_j$, is chosen at the switching time, the re-initializable adaptive model $M_I$ is the same as $M_j$ at that instance, with the same identification error, $e_I(t) = e_j(t)$, and performance index, $\Phi_I(t) = \Phi_j(t)$. Thereafter, $M_I$ is left to adapt until the next switching instance or when a transition is detected.

This type of adaptive model does not affect the previous discussed switching scheme. The stability of the combination of nonadaptive and adaptive models is the same as that guaranteed by theorem 2.

4 MIMO Nonlinear Chemical Reactor

The process, as developed by Ricker [7], consists of a single reactor whose total volume is a function of the operating conditions and the product flow rate. A single irreversible reaction

$$A(g) + C(g) \rightarrow D(\ell)$$

occurs in the vapor phase to produce $D$ which is non-volatile and is the only liquid component in the process. All the other components are non-condensible gases. The reaction rate depends only on the partial pressures, $P_j$, of the two reacting gases.

As Figure 1 illustrates, feed 1 contains components $A$, $B$ and $C$, with $B(g)$ described as an inert component. Feed 2 contains only component $A$, and is 40 times smaller as compared to feed 1. It
is used primarily to compensate for disturbances in the \( A/C \) ratio of feed 1. The solubility of \( A, B, \) and \( C \) in \( D \) is negligible. The product rate, stream 4, is adjusted by proportional feedback control (not shown) in response to variations in the liquid level. The purge rate, stream 3, depends on the pressure in the reactor, and can be manipulated to control the reactor pressure.

The process instrumentation is as follows. All the flow rates, \( F_i, (i = 1, 2, 3) \), the pressure, \( P \), and liquid level, \( V_L \), are measured. Measurements of the purge composition, \( y_{j3} \), are available at 6 minute sampling intervals. There are active constraints which include, keeping the reactor pressure below the shutdown limit of 3000 kPa, and saturation constraints on the flow rates.

A mathematical model, provided by Ricker, is given as

\[
\begin{align*}
\frac{dN_A}{dt} &= y_{A1}F_1 + y_{A2}F_2 - y_{A3}F_3 - r_D \\
\frac{dN_B}{dt} &= y_{B1}F_1 - y_{B3}F_3 \\
\frac{dN_C}{dt} &= y_{C1}F_1 + y_{C2}F_2 - y_{C3}F_3 - r_D \\
\frac{dN_D}{dt} &= r_D - F_4 \\
r_D &= k_0P_A^{\nu_1}P_C^{\nu_2} \\
V_L &= \frac{N_D}{\rho_L} \\
F_i &= \chi_iF_{i,\text{max}} \quad i = 1, 2 \\
F_i &= \chi_i\nu_i\sqrt{(P - 100)} \quad i = 3, 4
\end{align*}
\]

where \( r_D \) is the rate law and \( N_j \) is the molar holdup of component \( j \). Other definitions and nominal values can be found in Table 1. It is also assumed that the ideal gas law is valid and that the liquid density is constant.

The following scenarios are provided to test the control strategy.

- **Scenario I**: Regulate a disturbance (7.2% decrease in \( y_{A1} \), 7.2% increase in \( y_{B1} \)) in \( F_1 \) while keeping the production rate within 5% of its nominal value.

- **Scenario II**: Increase the production rate by 30% as rapidly as possible while maintaining all the process variables within their constraints.

- **Scenario III**: Regulate the process variables in the face of a linear drift (14.5% decrease in \( k_0 \), 12.5% decrease in \( \nu_2 \)) that occurs over a 48 hour period in the kinetic parameters.
There are a total of 10 output variables to be controlled but only 4 manipulated variables. Following Ricker the controlled variables are selected according to the objectives of the process [7]. \( F_4 \) is the production rate and should be controlled. The sensitivity of the reaction on reactor pressure is very high, therefore it should be controlled.

At steady state conditions, \( F_4 \) is equal to \( r_D \). Therefore, once \( F_4 \) and \( P \) are specified, \( y_{A3} \) and \( y_{C3} \) are related by the rate equation. It then follows that it is enough to specify only one of them. Since the reaction rate is more sensitive to component \( A \), \( y_{A3} \) is the logical component to be controlled. Care should be taken to specify a feasible set of values for \( F_4, P \), and \( y_{A3} \).

Notice that once the liquid level and the pressure are specified, the total number of moles, \( N \), in the gaseous phase is also specified by the ideal gas law. The natural manipulated variable to control the reactor liquid level is the reactor bottom stream, \( F_4 \). The valves’ responses are modeled as first order processes with time constants of 6 minutes, which is very small compared to the open-loop time constants.

Although the process is MIMO, a SISO multiple model reference adaptive structure (MMRAC) is being applied. Therefore, pairing of the inputs and outputs must be found that results in the best pairings with a minimum of interactions. RGA (Relative Gain Array) analysis, although a steady state tool, can be used to pair controlled and manipulated variables [23]. The RGA was established by making changes around the nominal operation using the model and determining the DC gains. The results of this analysis strongly suggests the following pairings, \( F_4 - F_1, y_{A3} - F_2, \) and \( F_3 - P \). The analysis also points to a strong dependence of \( P \) on \( F_1 \).

If \( F_4 \) is controlled by \( F_1 \), when there is a production increase, \( F_1 \) will increase and so will the reactor pressure. To maintain the pressure below the shutdown limits, the manipulated variable, \( F_3 \) must have a wide range to respond. However, based on the given design, the purge valve is not sized to
handle large and fast increases in pressure. To address this, a dynamic decoupler can be designed, but this is not the only alternative. Indeed, an override loop (see Figure 2a) is used in the multiple PI loop design [7] that adjusts the set point on the production rate should the rate of increase in the pressure be near its alarm limit (2900 kPa). Thus, production is sacrificed for safety.

In the case of MMRAS, models will be identified for the $F_4 - F_1$ and $P - F_3$ loops. The loop $y_{A_3} - F_2$ loop will be controlled using a PI controller because its effect on the reactor performance is much less in comparison to $P$ and $F_4$. To account for the interaction between $F_1$ and $P$ an adaptive model and controller pair will also be identified (see Figure 2b). In the MMRAS strategy, the change in $F_1$ is based on a ratio (5.6:1) between loops $P - F_1$ and $F_4 - F_1$ to minimize changes to the production rate while regulating the pressure.

4.1 Reference Models

The basic concept of any model reference strategy is to make the process output track a reference output. The input signal, $r(t)$, to the reference model can be any constant or piecewise continuous function. The choice of the reference model, $W_r$, should exhibit behavior that is neither too aggressive nor sluggish [8, 9]. It is not unusual to choose the closed-loop time constant to be faster than that of the open-loop time constant and for the order of the numerator and the denominator polynomials to be the same order as the identified models. Observing the above specifications any linear, stable, non-minimum phase reference model can be chosen. These choices are somewhat arbitrary, and the results obtained are closely linked to the choices made.

For a production rate change, a ramp like response is practical as oppose to a step change. The following reference model is chosen for the $F_4 - F_1$ loop,

$$\frac{F_{4r}(z)}{F_{1r}(z)} = \frac{0.0764}{z(z - 0.9236)}. \quad (18)$$
Observe that one pole is placed at zero (Z-domain) while the other is selected to yield a stable response.

Rapid increases in the production rate means an increase in both $F_1$ and $P$. Since the purge valve is undersized, operating the process safely is a primary concern. The reference model chosen for the $P - F_1$ loop is given by

$$\frac{P_r(z)}{F_{1r}(z)} = \frac{-1.3(z - 0.834)}{z^2 - 0.6718z - 0.1124}. \quad (19)$$

In practice, it is always advisable to filter the reference signal to prevent overly aggressive control action. For scenario II, a first order filter with a time constant of 1.1 and 2 hours are used with the $F_{4r} - F_{1r}$ and $P_r - F_{1r}$ loops, respectively. The filter gains in both cases are unity. The choice of the filter time constants is based on balancing production demand with satisfactory regulation of the reactor pressure. No attempt was made to optimize the choice of the filter time constants.

For scenarios I and III, deviations from the nominal must be compensated. Since all the models are in deviation variables, any changes in the process and reference outputs must be kept small. However, for the parameters to adapt, enough excitation in the reference signal must be present [8]. As such, $r(t)$ is chosen to be a small random signal (dither signal) with mean 0 and variance 0.01.

4.2 Model Identification

The multiple model reference adaptive structure requires models of the control loops, $F_4 - F_1$, $P - F_3$, and $P - F_1$. The identified models may consist of nonadaptive and adaptive models depending on the quality of the response desired (stability, accuracy and speed). It is intuitive that these models are identified near known operating regions of the plant or distributed uniformly over the operating space [24]. In practice, only the nominal operation and set point transitions are
known in advance. However, in this specific case, since pressure is a concern, nonadaptive models in the range of 2700 to 2900 kPa can be identified to obtain a stable and fast response.

Models for the loop-pairings must be identified around the nominal operation. The following models were identified by making a small step change after the process reached steady-state at one of the nominal conditions. These data were analyzed using the Matlab® System Identification Tool Box by Mathworks (Natick, MA) [25].

For the model of loop $F_4 - F_1$, at the nominal condition, a second-order discrete input-output model is identified as,

$$
\frac{F_4(z)}{F_1(z)} = \frac{0.1641}{z^2 - 0.9432z + 0.0444}.
$$

The pressure response to step changes in $F_3$ at the nominal condition, contains both a fast and a slow integrating response. Each one can be thought of as a first order process and together they represent the overall response. A first-order transfer function is identified for each giving rise to the following Z-domain model,

$$
\frac{P(z)}{F_3(z)} = \frac{-0.4652(z - 0.9713)}{z^2 - 1.762z + 0.7635}.
$$

Observe that the gain is negative as expected since opening the purge valve causes the pressure to decrease.

The response of pressure to a step change in $F_1$, at the nominal condition, exhibits two distinct time constants (10 and 0.25 hours) that are a magnitude apart. Here as well the overall response is approximated by the sum of two first order responses,

$$
\frac{P(z)}{F_1(z)} = \frac{10.2006(z - 0.9824)}{z^2 - 1.59z + 0.5940}.
$$

Figures 3 - 5 illustrate the process (\textasteriskcentered) and model (\texttimes) responses of the $F_4 - F_1, P - F_1$, and $P - F_3$ loops, respectively. It is observed that the fit of the model obtained for the $F_4 - F_1$ loop is very
accurate whereas those obtained for the other loops are less so. Improvements in the model’s accuracy did not improve their performance. All the identified models are strictly proper and stable even though one root of loops, \( P - F_1 \) and \( P - F_3 \), lies very close to the margin of instability. For stable parameter adaptation, the sign of the open-loop gain should not change. For this process, the sign of the gains for each loop remains unchanged throughout the operating space.

Two additional sets of nonadaptive models are similarly identified; one at the known production rate change (\( F_4 = 130 \) kmol/hr, \( P = 2850 \) kPa, and \( y_{A3} = 63\% \)) and a third is arbitrarily placed in the operating space. The general second order model form is given by

\[
W_p(s) = \frac{k_p}{s^2 + \frac{b_1}{b_0}s + a_1}
\]

with \( k_p = b_0 \). Table 2 lists the parameters of the nonadaptive models and their companion controllers.

Using this collection of reference and nonadaptive models, the performance of the multiple model reference strategy is tested. Simulation results show that although stable and fast responses are obtained in all scenarios they lacked accuracy due to the absence of adaptation. For instance, Figure 6 represents the responses of production rate (Figure 6a) and reactor pressure (Figure 6b) for scenario II. The reference output is denoted by * and the nonlinear plant output by the solid line.

When free-running adaptive models, one for each loop initialized at the nominal conditions, are added to the set of models improved accuracy in the responses are obtained. However, if the disturbance originates at conditions other than the nominal, large transient errors are more likely to occur due to a combination of slow adaptation and a scarcity of nonadaptive models. The inclusion of a re-initializable adaptive model can further improve the MMRAS performance.

Figure 7 compares the responses of the production rate and reactor pressure obtained for scenario
III, when the set of models contain nonadaptive and re-initializable adaptive models or nonadaptive, re-initializable and free-running adaptive models. It is observed that very little improvement (the responses are almost indiscernible) in the performance (accuracy, speed, and stability) is gained when both types of adaptive models are included. As a result, the performance of a model set that consists of nonadaptive (see Table 2) and re-initializable adaptive models and their controllers will be applied to control the MIMO nonlinear chemical reactor.

4.3 Tuning

The tuning rules were developed for unconstrained SISO LTI noninteractive processes [24]. Since the parameters of the adaptive models and controllers are being re-tuned at each opportunity, aggressive control action may occur resulting in violation of active constraints. For example, in scenario II, the tuning rules when used without any modification may lead to very rapid changes and unwanted oscillations in the manipulated variable. For this system, adaptive gains ($< 1$ but positive for stable tuning) are used to slow the rate of adaptation. The adaptive gains used with the re-initializable adaptive models are $0.6$ for the $F_4 - F_1$ loop and $0.06$ for the $P - F_3$ and $P - F_1$ loops. No attempt was made to optimize these values.

4.4 Switching

Switching between multiple controllers requires logic to decide which controller to switch to and when to do so [13, 16]. The model whose controller gives the least control error is preferred but the performance of a controller cannot be evaluated until its output is implemented. Therefore, it is not possible to select the best controller based on the smallest control error. Since all the identification models are in parallel with the process and share the same input, it follows that a performance index that is a function of the identification error of the models can be used to guide
the selection of the controller.

The performance index used is Equation (16)

\[ J_k(t) = v_1 e_k^2(t) + v_2 \int_0^t e^{-\lambda(t-\tau)} e_k^2(\tau) d\tau \]

with \( v_1, v_2 > 0 \), and \( \lambda \) the forgetting factor, chosen to reduce the influence of the past errors. The integral term represents a weighted sum of all the past errors. The parameters \( v_1 \) and \( v_2 \) are chosen to give more emphasis on the past rather than the present errors. If a large value of \( \lambda \) or a lower value of \( v_2 \) is chosen, then the current identification error has greater influence on the performance and may lead to unnecessary switching. On the other hand if greater emphasis is given to the past errors the switching may be too conservative and the control action may lose both speed and accuracy.

If the switching time or the time between two successive switches is too fast, this may lead to chattering. On the other hand if either is too slow, the current controller in parallel with the plant may not be the best choice during the elapsed time prior to switching to another controller. The switching time used for all the scenarios is 0.1 hours. A good overview of several switching schemes is provided in ref [13].

4.5 Results

The simulation results obtained using MMRAS will be compared to the multiple PI (MPI) controller strategy developed by Ricker [7]. In all the figures, the * represents the reference output, the solid line and \( \triangle \) represent the performance of the nonlinear plant output that results from applying MMRAS and MPI, respectively. The models are indexed as follows, indices 1-3 are nonadaptive models and index 4 is the re-initializable adaptive model.
4.5.1 Scenario I

In the face of this unmeasured feed composition disturbance the goal is to maintain the production rate to within 5% of the nominal value (100 kmol/h). The MMRAS production rate response (see Figure 8) shows an initial decrease for 5 hours followed by an increase ($t=12$ h) and return to its nominal value. The minimum production rate is 91.8 kmol/h. The MPI production rate response is qualitatively similar to MMRAS with a minimum production rate of 93 kmol/h. Both responses have not settled even after 30 hours. Using the integral of the square error (ISE) as a measure of the difference between the response and its nominal value, the MPI gives a smaller loss in production as compared to MMRAS.

Figure 9 shows the reactor pressure performance. In the MMRAS case, the pressure increases reaching a maximum of 2800 kPa because the purge valve has saturated. The response eventually settles at the nominal value after 20 hours. In the MPI strategy, the pressure increases to a maximum of 2850 kPa, coming to within 50 kPa of the alarm limit, before returning to the nominal value after 30 hours. The purge valve response in both cases is similar. Again using the ISE, the MPI pressure response deviation is 46 times greater than the MMRAS response. Clearly, the smaller production loss by the nonlinear process when controlled by the MPI scheme is achieved at the expense of a pressure violation. In the case of the MMRAS, production is sacrificed to maintain safe pressure operation.

Figure 10 shows the switching performance of the MMRAS. Initially, some switching is observed among the nonadaptive and the re-initializable adaptive model. As the identification error between the re-initializable adaptive model and the process decreases (and indirectly the controller output error), the switching logic settles down to a choice of the re-initializable model.
4.5.2 Scenario II

An increase in the production rate by 30\% is required. The production rate and reactor pressure reference outputs are given by Equations (18) and (19), respectively. Figure 11 shows the production rate response. It is observed that satisfactory tracking of the reference output is achieved with the desired set point reached in 10 hours by the MMRAS. In the MPI strategy, the production rate initially rises monotonically to 115 kmol/h, but due to the rapid increase in the reactor pressure and saturation of the purge valve, the override loop decreases the production rate set point and the actual production rate drops. Eventually, the production rate set point is returned to the desired value and the production rate asymptotically converges to this value. The production loss by MPI is 1.3 times greater as compared to MMRAS.

The performance of the reactor pressure is shown in Figure 12. Satisfactory tracking of the reference output is obtained in the MMRAS case; the resulting response does not exhibit any overshoot and settles after 10 hrs. In the MPI case, the pressure exhibits an overshoot of 27\% and comes to within 10 kPa of the alarm limit.

4.5.3 Scenario III

A drift in the kinetic parameters occurs that is linear over a 48 hour period. The production rate cannot be maintained at the nominal value as it is limited by the rate of reaction. Figure 13 shows the response of production rate of the nonlinear process. The adaptive controller on the $F_1 - F_1$ loop responds to the decrease in the production rate by increasing the rate of feed 1. However, this causes a rapid increase in the reactor pressure which cannot be regulated by the purge valve. The adaptive controller associated with the $P - F_1$ loop responds by decreasing the rate of feed 1 to maintain safe operation of the reactor. In the MPI strategy, a similar response is observed but the
loss in the production rate is less because the pressure is at the high alarm limit (Figure 14) and actually exceeds it when \( t = 40 \) hours. In contrast, the pressure response by MMRAS stays well below the high alarm limit. Compared to the MMRAS, the MPI strategy preferentially chooses the production goal over safety. The loss in production is 7.2 times greater in the MMRAS response but safe operation is maintained.

### 4.6 Summary and Discussion

The results and performance of the multiple model reference adaptive structure can be summarized as follows. The identified linear models of the nonlinear chemical reactor are completely controllable and observable. The stability analysis that was presented in §3 was developed for LTI but not necessarily stable SISO systems. However, when the process is known to be open-loop stable, stability ceases to be the main issue and the MMRAS can be used to improve the speed and accuracy of the performance. The simulation results support this conclusion in spite of the nonlinear, multivariable, and interactive nature of the reactor with active constraints.

In all scenarios, the parameters of the adaptive models and their controllers converge although not to the true values. This is not surprising as it is a well known fact that satisfactory control can be achieved regardless of this limitation. It is also observed, but not shown, that the identification errors between the outputs of the re-initializable models and the outputs of the nonlinear process asymptotically approach zero in the limit as \( t \to \infty \). In turn the error in the controller output asymptotically approaches zero. The selection of the adaptive model over the nonadaptive ones was also demonstrated by the switching logic.

The response obtained using MMRAS is faster when compared to MPI. In a decentralized model predictive control strategy developed by Ricker, the production rate and reactor pressure reach
their set points quickly in scenario II, but exhibit large overshoots [7]. Additionally, the MMRAS was designed to give greater importance to regulating the reactor pressure rather than maintaining production rate. However, a higher production rate can be obtained by changing the ratio (5.6:1) between the $F_4 - F_1$ and $P - F_1$ loop.

5 Conclusions

The use of multiple models and controllers is attractive especially when the process is known to transition to unknown operating states. It is also intuitive that a fixed parameter controller may not provide satisfactory closed-loop control. The multiple model concept can be made more responsive if both adaptive and nonadaptive models are used. The nonadaptive models can provide speed whenever its parameters are close to those of the process while the adaptive models can provide accuracy because its parameters are permitted to adapt. Two types of adaptive models were discussed, free-running and re-initializable models. In the former, the adaptation begins from the current parameter set, in the latter the adaptation begins with the parameters of the nonadaptive model whose identification error is the smallest.

Two theorems about stability for single-input single-output, linear time invariant systems were provided when the model set is either nonadaptive or a combination of adaptive and nonadaptive models. Stability it appeared largely rests with the selection of a minimum elapsed time before switching to another controller is allowed.

The multiple model reference adaptive structure that employed single-input single-output adaptive and nonadaptive models and controllers was successfully demonstrated on a multiple-input multiple-output constrained, nonlinear but stable chemical reactor in the presence of unmeasured disturbances, parametric uncertainties, and a product rate transition.
Many issues remain that must be investigated before the multiple model reference adaptive structure can be generalized to address plant-wide control problems and multiple-input multiple-output processes with a greater degree of nonlinear, interactive behavior. These include, selection of the reference models, robust performance criteria, and nonlinear stability analysis. These topics are currently being investigated.

**Acknowledgements:** The authors would like to acknowledge the financial support provided by NSF Grant CTS-0096024.
Literature Cited


Table 1: Chemical reactor parameters and nominal conditions

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Table 2: Parameters of the nonadaptive models and controllers.

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Figure 1: Schematic representation of the 2-phase reactor
Figure 2: (a) Pressure control using override PI controller strategy that modifies the production rate set point. (b) Pressure control using adaptive controllers that modifies feed 1 flow rate.
Figure 3: The production rate response to a step change in feed 1. •: process response, ×: identified model response.
Figure 4: The pressure response to a step change in feed 1. *: process response, ×: identified model response
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Figure 8: The production rate response for scenario I. Solid line: nonlinear process output using MMRAS, *: reference model output, and Δ: nonlinear process output using MPI.
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Figure 13: The production rate response for scenario III. Solid line: nonlinear process output using MMRAS, *: reference model output, and Δ: nonlinear process output using MPI.
Figure 14: The pressure response for scenario III. Solid line: nonlinear process output using MMRAS, *: reference model output, and Δ: nonlinear process output using MPI.