A State-Shared Model Predictive Control Approach to Transition Control

Zhenhua (David) Tian and Karlene A. Hoo
Department of Chemical Engineering
Texas Tech University, Lubbock, TX
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Objectives

Assume the operating space is large and there are many operating points.

- Given the transfer functions of all known operating points - develop one non-minimal realization.
- At any other point - develop a non-minimal identifer with the same structure as the non-minimal realization.
- Non-minimal realization OR non-mininal identifier:
  state-shared model
  All models share the same state-space representation
- Model-based control
  State-shared model $+$ measurement equations
- Application
  MIMO nonlinear chemical reactor

MIMO linear system properties

Theorem 1 H(s) is realizable as the transfer function matrix of a linear time-invariant system given by

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

if and only if H(s) is a proper rational matrix. If D is the null matrix, then H(s) is a strictly proper rational matrix.

$H(s)$ can be written as a matrix fraction description,

$$H(s) = D^{-1}(s)N(s), \qquad D(s) = d(s)I_p$$

where $d(s) = s^n + d_1 s^{n-1} + \cdots + d_n$, is the least common multiple of the denominators of $H(s)$ and degree of $d(s)$, deg $d(s) = n$

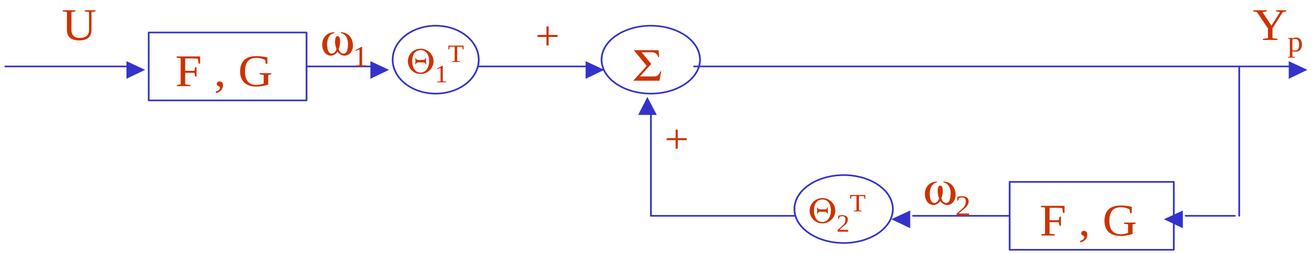
Lemma 1 (Row degree of Proper Transfer Functions) If H(s) is a strictly proper (proper) transfer function and the left MFD is given by

$$H(s) = D^{-1}(s)N(s)$$

then every row of N(s) has degree strictly less than (less than or equal to) that of the corresponding row of D(s).

Kailath, Linear Systems, 1980
Antsaklis and Michel, Linear Systems, 1997

Non-minimal realization



Theorem 2 Any controllable and observable m-input m-output LTI system is input-output equivalent to the LTI system described by the differential equations

$$\dot{\omega}_1 = \mathbf{F}\omega_1 + \mathbf{G}\mathbf{U} \qquad \omega_1 \in R^{nm\times 1}$$

$$\dot{\omega}_2 = \mathbf{F}\omega_2 + \mathbf{G}\mathbf{Y}_\mathbf{p} \qquad \omega_2 \in R^{nm\times 1}$$

$$\mathbf{Y}_\mathbf{p} = \boldsymbol{\Theta}^\mathbf{T}\omega \qquad \text{measurement}$$

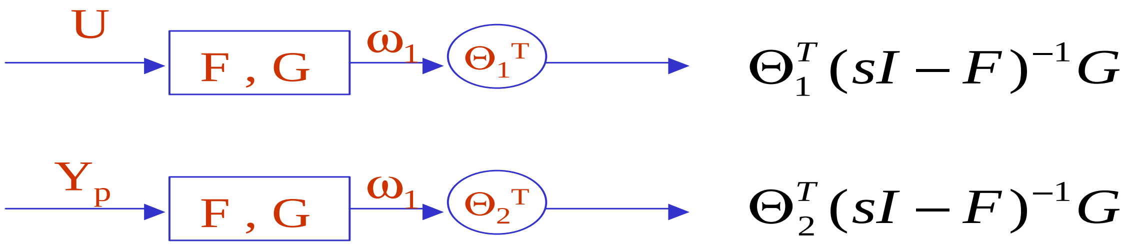
by suitable choice of the parameter vector $\Theta \in \mathrm{R}^{2\mathrm{nm}\times\mathrm{m}}$. The pair (F,G) should be controllable, $\mathrm{F} \in \mathrm{R}^{\mathrm{nm}\times\mathrm{nm}}$ is an asymptotically stable matrix, and $\mathrm{G} \in \mathrm{R}^{\mathrm{nm}\times\mathrm{m}}$.

Narendra & Annaswamy, Stable Adaptive Systems, 1989

Similar theorem for SISO systems.

Uniqueness of model parameters

Corollary 1 Given $\mathrm{N(s)}$ and $\mathrm{D(s)}$, the parameter vector $\Theta^{\mathrm{T}} = [\Theta_1^{\mathrm{T}}\ \Theta_2^{\mathrm{T}}]$ $(\Theta_1, \Theta_2 \in \mathrm{R}^{\mathrm{nm}\times\mathrm{m}})$ exists and is uniquely determined.



$$\mathrm{Y_p(s)} = \Theta_1^{\mathrm{T}}(\mathrm{sI}-\mathrm{F})^{-1}\mathrm{G\,U(s)} + \Theta_2^{\mathrm{T}}(\mathrm{sI}-\mathrm{F})^{-1}\mathrm{G\,Y_p(s)}$$

$$\Phi_{\mathrm{I}} \equiv (\mathrm{sI}-\mathrm{F})^{-1}\mathrm{G}$$

$$\mathrm{U(s)} \rightarrow \mathrm{Y_p(s)} = \mathrm{H(s)} = \left(\mathrm{I} - \Theta_2^{\mathrm{T}}\Phi_{\mathrm{I}}\right)^{-1}\left(\Theta_1^{\mathrm{T}}\Phi_{\mathrm{I}}\right)$$

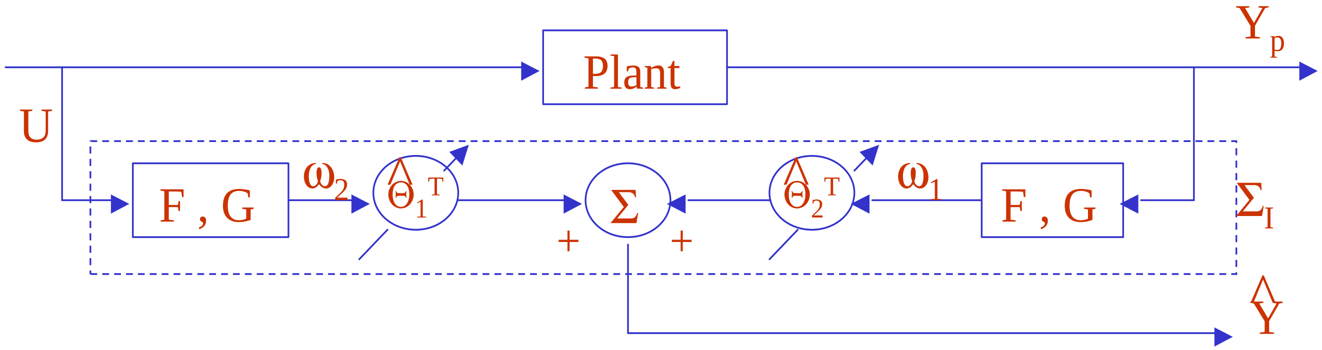
$$\Omega_{\mathrm{I}}(\mathrm{s}) \equiv \det(\mathrm{sI}-\mathrm{F}) = \mathrm{s}^{\mathrm{n}} + \mathrm{a_n s^{n-1}} + \cdots + \mathrm{a_2 s} + \mathrm{a_1}$$

$$\mathrm{H(s)} \equiv \mathrm{D^{-1}(s)N(s)} = (\mathrm{D(s)}/\Omega_{\mathrm{I}}(\mathrm{s}))^{-1}(\mathrm{N(s)}/\Omega_{\mathrm{I}}(\mathrm{s}))$$

Equivalently

$$\mathrm{I} - \Theta_2^{\mathrm{T}}\Phi_{\mathrm{I}} = \frac{D(s)}{\Omega_I(s)} \qquad \Theta_1^{\mathrm{T}}\Phi_{\mathrm{I}} = \frac{N(s)}{\Omega_I(s)}$$

Non-minimal identifier



Identify the measurement equation parameters $\hat{\Theta}$

Measured signals: U and Y_p

Non-minimal realization form

Define

Parameter errors	$\tilde{\Theta} \equiv \hat{\Theta} - \Theta$
Model errors	$\tilde{Y} \equiv \hat{Y} - Y_p$
Lyapunov function	$V = \frac{1}{2}\tilde{\Theta}^T\tilde{\Theta} > 0$

It can be proven

$$\lim_{t\to\infty} \tilde{Y}(t) = 0, \qquad \lim_{t\to\infty} \hat{\Theta}(t) = \Theta$$

Adaptation convergence analysis

Barbălat's Lemma: if the infinity norm of a uniform continuous function is bounded then the function will asymptotically approach 0.

$$\lim_{t\to\infty} \int_0^t |f(\tau)| d\tau < \infty$$

$$\lim_{t\to\infty} f(t) = 0$$

Proposition 1 If $g \in \mathcal{L}^\infty$ and $\dot{g} \in \mathcal{L}^\infty$, then g^2 is uniformly continuous.

Corollary 2 If $g \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, and $\dot{g} \in \mathcal{L}^\infty$, then $\lim_{t\to\infty} g(t) = 0$

Parameter adaptation

MIMO normalized least-squares

$$\dot{\theta}_1 = -g\frac{P\omega e_1}{1+\gamma\omega^T P\omega} \qquad g,\gamma > 0$$

$$\dot{\theta}_2 = -g\frac{P\omega e_2}{1+\gamma\omega^T P\omega} \qquad g,\gamma > 0$$

$$\cdots \qquad \cdots$$

$$\dot{\theta}_m = -g\frac{P\omega e_m}{1+\gamma\omega^T P\omega} \qquad g,\gamma > 0$$

$$\dot{P} = -g\frac{P\omega\omega^T P}{1+\gamma\omega^T P\omega} \qquad P(0) > 0$$

$\hat{\mathbf{\Theta}}_\mathbf{j} = [\theta_\mathbf{1}, \theta_\mathbf{2}, \cdots, \theta_\mathbf{m}]_{\mathbf{nm\times m}}$: **parameter matrix**

$\tilde{\mathbf{Y}}_\mathbf{j} = [\mathbf{e_1}, \mathbf{e_2}, \cdots, \mathbf{e_m}]^\mathbf{T}_{\mathbf{m\times 1}}$: **model-plant mismatch.**

State-shared model (SSM)

Non-minimal realization or non-minimal identifier = SSM

- State-shared state space model for MIMO system:

$$\dot{\omega}_1 = F\omega_1 + GY_p$$

$$\dot{\omega}_2 = F\omega_2 + GU$$

- All models share the same states - $\omega(t)$
- Driven by measurement signals: Y_p, U
- Measurement equation: different parameter values

$$Y_j = \hat{\Theta}_{1j}^T\omega_1 + \hat{\Theta}_{2j}^T\omega_2$$

$$\hat{\Theta}_j \equiv [\hat{\Theta}_{1j}^T,\ \hat{\Theta}_{2j}^T]^T$$

$$j = 1, \ldots, N$$

A selection of pair (F, G)

Let F be chosen as

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & & & & & & & & & \\ 0 & 0 & 1 & \cdots & 0 & & & & & & & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & & & & & & & & & \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n & & & & & & & & & \\ & & & & & \cdots & & & & & & & & \\ & & & & & & \ddots & & & & & & & \\ & & & & & & & \cdots & & & & & & \\ & & & & & & & & 0 & 1 & 0 & \cdots & 0 \\ & & & & & & & & 0 & 0 & 1 & \cdots & 0 \\ & & & & & & & & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & & & & & -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}_{nm \times nm}$$

A selection of pair (F, G)

Let G be chosen as

$$\begin{bmatrix}
0 & & & & \\
0 & & & & \\
\vdots & 0 & \cdots & & \\
1 & & & & \\
 & 0 & & & \\
 & 0 & & & \\
0 & \vdots & 0 & \cdots & \\
 & 1 & & & \\
\cdots & \cdots & \cdots & & \\
 & & & & 0 \\
 & & 0 & & 0 \\
 & & & & \vdots \\
 & & & & 1
\end{bmatrix}_{nm \times m}$$

$\mathbf{\Phi_I \equiv (sI - F)^{-1}G}$

$$\frac{1}{s^n + a_n s^{n-1} + \cdots + a_2 s + a_1}\left[\begin{array}{ccccc} \begin{matrix}\mathbf{1}\\ s\\ \vdots\\ \mathbf{s^{n-1}}\end{matrix} & 0 & \cdots & & \\ 0 & \begin{matrix}\mathbf{1}\\ \mathbf{s}\\ \vdots\\ \mathbf{s^{n-1}}\end{matrix} & 0 & \cdots & \\ \cdots & \cdots & \cdots & & \\ & & 0 & & \begin{matrix}\mathbf{1}\\ \mathbf{s}\\ \vdots\\ \mathbf{s^{n-1}}\end{matrix} \end{array}\right]_{\mathbf{nm \times m}}$$

State-shared model parameters: Θ

$$\Theta_1^T \equiv \begin{bmatrix} \theta_{1_{1,1}} & \theta_{1_{1,2}} & \cdots & \theta_{1_{1,nm}} \\ \theta_{1_{2,1}} & \theta_{1_{2,2}} & \cdots & \theta_{1_{2,nm}} \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{1_{m,1}} & \theta_{1_{m,2}} & \cdots & \theta_{1_{m,nm}} \end{bmatrix}_{m \times nm}$$

$$\Theta_2^T \equiv \left[\begin{array}{cccccccccccc} \theta_{2_1} & \theta_{2_2} & \cdots & \theta_{2_n} & & & 0 & & & & & \\ & & 0 & & \theta_{2_1} & \theta_{2_2} & \cdots & \theta_{2_n} & & 0 & & \\ & & \cdots & & \cdots & & \cdots & & \cdots & & & \\ & & & & 0 & & & & \theta_{2_1} & \theta_{2_2} & \cdots & \theta_{2_n} \end{array}\right]_{m \times nm}$$

The plant transfer function

$$\mathbf{N}(s) \equiv \begin{bmatrix} \sum_{i=1}^{n} k_{1,i}s^{i-1} & \sum_{i=n+1}^{2n} k_{1,i}s^{i-(n+1)} & \cdots & \sum_{i=n(m-1)+1}^{nm} k_{1,i}s^{i-(n(m-1)+1)} \\ \sum_{i=1}^{n} k_{2,i}s^{i-1} & \sum_{i=n+1}^{2n} k_{2,i}s^{i-(n+1)} & \cdots & \sum_{i=n(m-1)+1}^{nm} k_{2,i}s^{i-(n(m-1)+1)} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{i=1}^{n} k_{m,i}s^{i-1} & \sum_{i=n+1}^{2n} k_{m,i}s^{i-(n+1)} & \cdots & \sum_{i=n(m-1)+1}^{nm} k_{m,i}s^{i-(n(m-1)+1)} \end{bmatrix}_{m\times m}$$

$$\mathbf{D}(s) \equiv \begin{bmatrix} s^n + \sum_{i=1}^{n} d_i s^{i-1} & 0 & & \\ 0 & s^n + \sum_{i=1}^{n} d_i s^{i-1} & & \\ & & \cdots & \\ & & & s^n + \sum_{i=1}^{n} d_i s^{i-1} \end{bmatrix}_{m\times m}$$

Parameter values

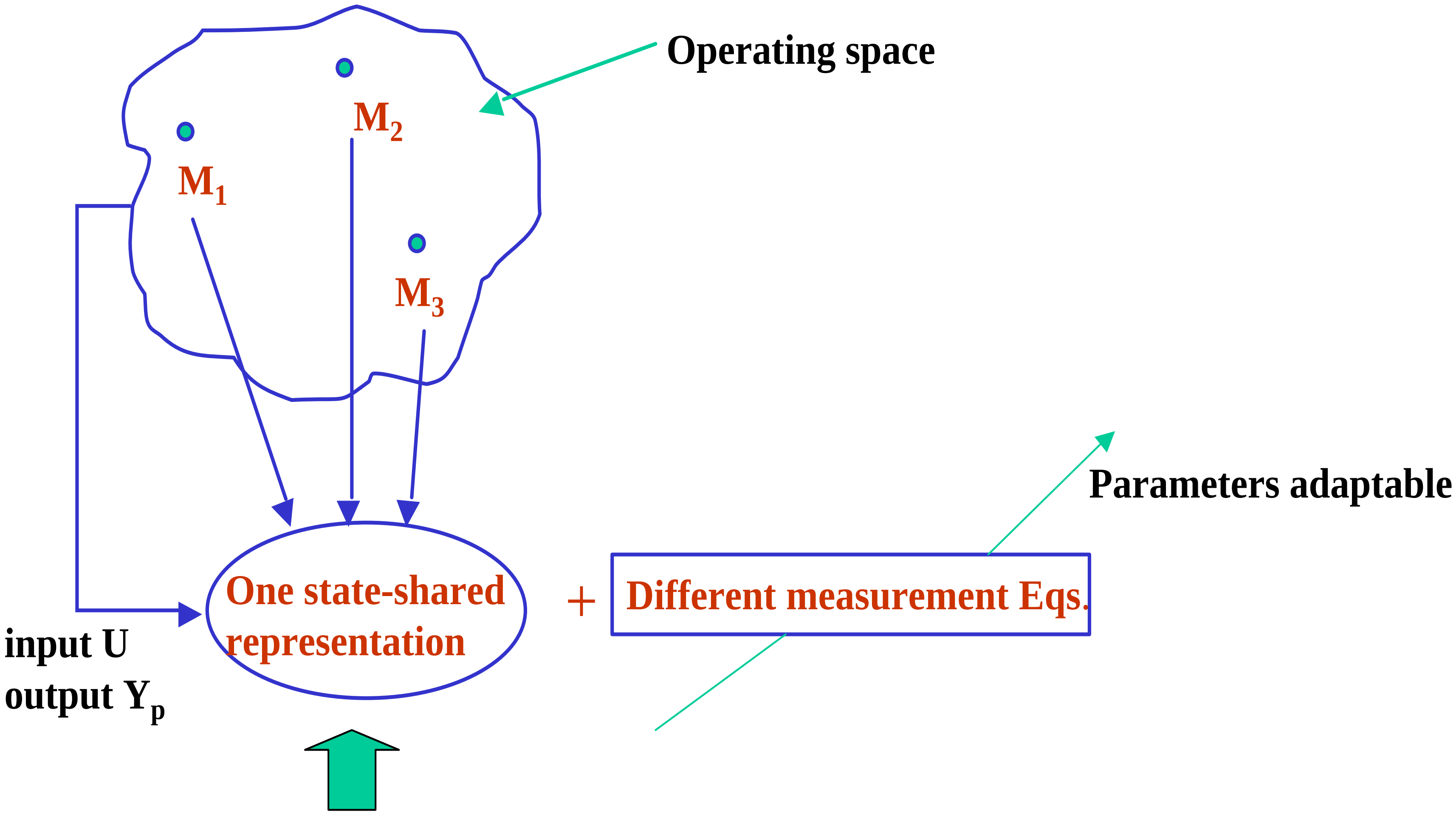
$$\theta_{1_{i,j}} = k_{i,j} \quad i = 1, \cdots, m;\ j = 1, \cdots nm$$

$$\theta_{2_i} = a_i - d_i \quad i = 1, \cdots, n$$

Application: Transition control in a large operating space

- Transition
  - Production rate change
  - Grade changeover
  - Disturbance or parameter change
  - Startup or shutdown
- Controller Performance
  - Stable response during the transition
  - Fast response
  - Accurate response
- Operability and Economic Performance
  - Reduce the amount of off-spec products
  - Minimize lost production capacity
  - Shorten transition time
  - Minimize utility costs

Transition control - Multiple models



Operating space

M_1

M_2

M_3

input U

output Y_p

One state-shared representation

+ Different measurement Eqs.

Parameters adaptable

Reduce the number of the models to a manageable size

Requirements on modeling

- Model (fixed) at each known operating state
  — non-minimal realization
- At any other point (transition) — non-minimal identifier
- Together — State-shared model (SSM)
- Controller should be implementable (low order)
  — Balanced truncation method (Burl, Linear Optimal Control, 1999).

Advantages

- No assumption that the fixed models cover the large operating space
- SSM: reduces the number of fixed models
- SSM: reduces computational burden

Model reduction

Definition 1 Balanced realization of state space model if the controllability and and observability grammians are the same and diagonal, i.e.,

$$\mathbf{W}_{\mathcal{C}} = \mathbf{W}_{\mathcal{O}} = \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_n \end{bmatrix}$$

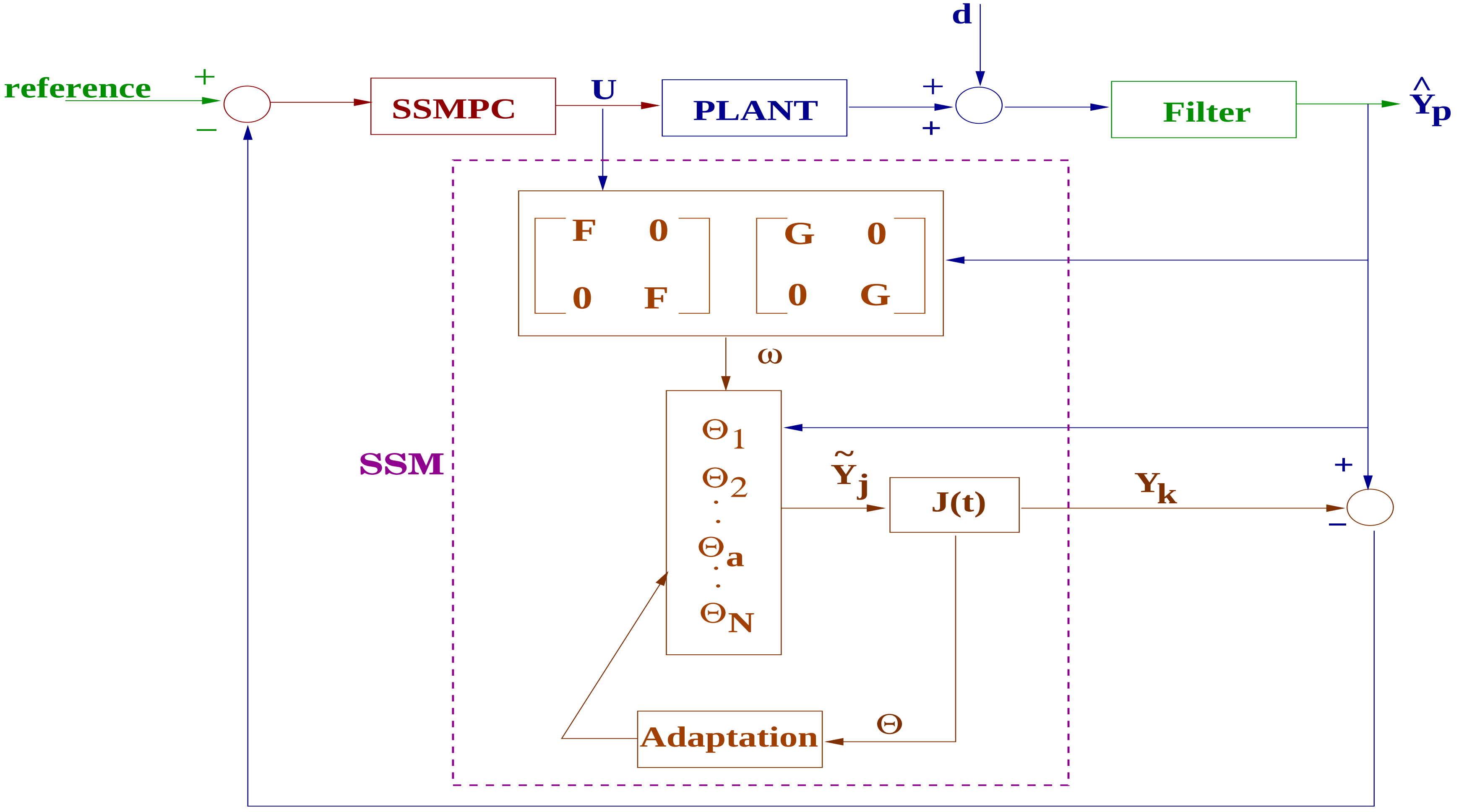
Partition the balanced realization as:

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{x}}_R(t) \\ \dot{\mathbf{x}}_2(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_R & \mathbf{A}_{12} \\ \mathbf{A}_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_R \\ \mathbf{B}_2 \end{bmatrix} \mathbf{u}(t) \\ \mathbf{y}(t) &= [\,\mathbf{C}_R \quad \mathbf{C}_2\,] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \end{aligned}$$

Balanced truncation eliminates those states in the balanced realization with the smallest elements in the grammians. Quantify the error in the approximation: (Burl, 99)

$$\|\boldsymbol{\Delta}(s)\|_\infty = \|\mathbf{G}_R(s) - \mathbf{G}(s)\|_\infty \leq 2(\sigma_{k+1} + \sigma_{k+2} + \cdots + \sigma_n)$$

SSM Predictive Control system

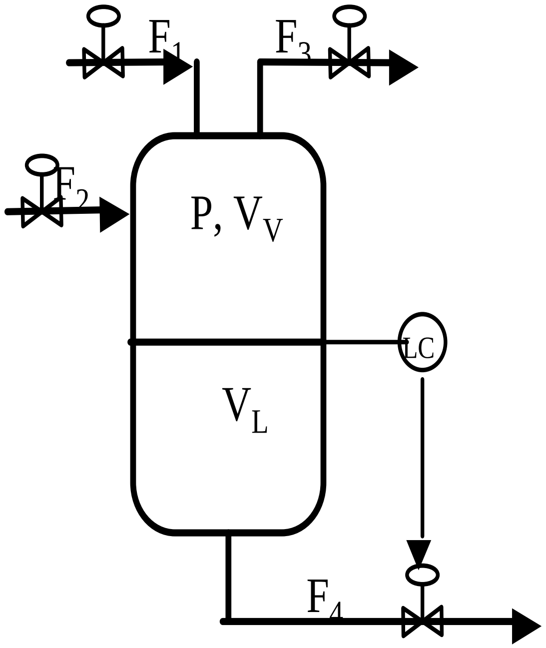


Performance index

$$J_j(t) = \mu \|\tilde{Y}_j(t)\|^2 + \nu \int_0^t \|\tilde{Y}_j(\tau)\|^2 d\tau, \qquad j = 1, \ldots, N$$

$\mu, \nu \geq 0$ user-specified parameters

Two-phase MIMO nonlinear reactor



A(g)+C(g)®D(l)
B: inert component

- Manipulated variables
  - F_1, Feed includes components A(g), B(g), C(g)
  - F_2, Pure component A(g)
  - F_3, Purge includes components A(g), B(g), C(g)
- Controlled variables
  - P, Pressure in reactor
  - y_{A3}, Concentration of A(g) in purge
  - F_4, Production D(l) flow rate

Ricker, J.Process Control,Vol3, No2, 1993.

Transition between two operating points

Variable	Definition	OPI	OPII
χ_1	**Valve 1**	60.95%	78.46%
χ_2	**Valve 2**	25.02%	47.68%
χ_3	**Valve 3**	39.25%	56.17%
χ_4	**Valve 4**	44.17%	59.41%
F_1	**Feed 1 rate**	**201.43 kmol/h**	**259.28 kmol/h**
F_2	**Feed 2 rate**	**5.62 kmol/h**	**10.72 kmol/h**
F_3	**Purge rate**	**7.05 kmol/h**	**10.37 kmol/h**
F_4	**Production rate**	**100 kmol/h**	**130 kmol/h**
P	**Reaction pressure**	**2700 kPa**	**2850 kPa**
V_L^*	**Setpoint of liquid level**	44.18% **of max**	44.18% **of max**
V_L	**Liquid level**	**13.254 m**^{3}	**15.907 m**^{3}
y_{A3}	**Mole fraction of A in purge**	**47.00 mol %**	**63.40 mol %**
y_{B3}	**Mole fraction of B in purge**	**14.29 mol %**	**12.02 mol %**

The full-order linear model at OPI

$$\mathbf{A} = \begin{bmatrix} -2.7361 & 0.0350 & -1.0565 & -1.4713 \\ 0.0106 & -0.0638 & 0.0106 & 0 \\ -2.6679 & 0.0288 & -1.1371 & -1.4713 \\ 0.2588 & -0.0661 & 0.0654 & -1.0758 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.4850 & 1.0000 & -0.4700 \\ 0.0050 & 0 & -0.1429 \\ 0.5100 & 0 & -0.3871 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 28.5171 & 28.5171 & 28.5171 & 24.8288 \\ 0.5598 & -0.4964 & -0.4964 & 0 \\ 0.5484 & 0.5484 & 0.5484 & 10.4002 \end{bmatrix}$$

$\lambda(\mathrm{A})$=(**-3.6077, -1.2647, -0.0745, -0.0659)**

The reduced-order linear model at OPI

$$\mathbf{A} = \begin{bmatrix} -0.0564 & 0.2618 \\ -0.1330 & -3.6939 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0.4062 & -3.0471 & -1.1310 \\ -3.7278 & -5.2885 & 3.4003 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 3.2755 & -7.3066 \\ -0.1357 & -0.0391 \\ -0.0114 & -0.2008 \end{bmatrix}$$

Upper bound of McMillian degree: n=2

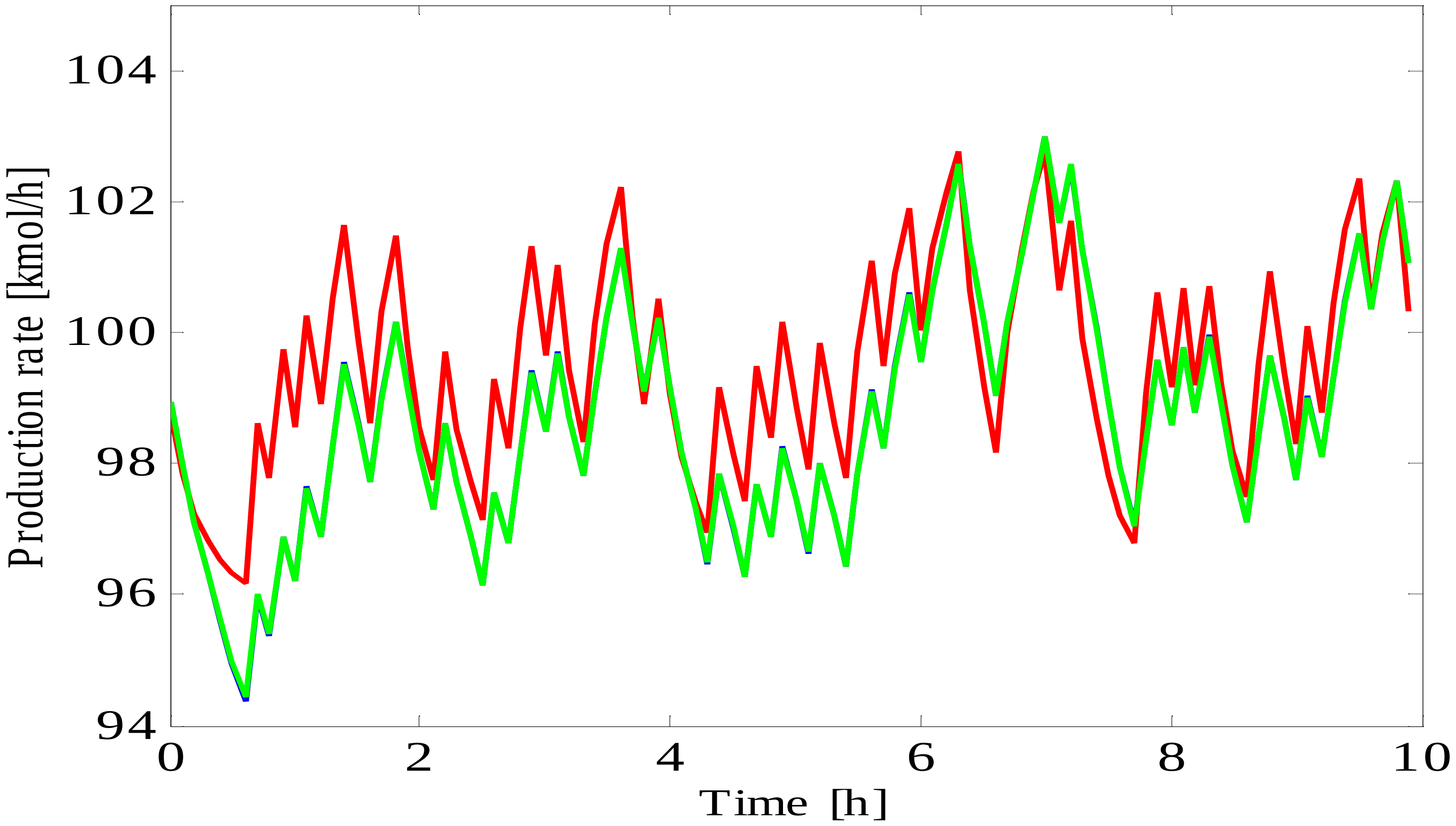
$$\mathbf{W}_{\mathcal{C}} = \mathbf{W}_{\mathcal{O}} = \boldsymbol{\Sigma} = \begin{bmatrix} 95.1559 & & & \\ & 7.2349 & & \\ & & 0.6913 & \\ & & & 0.2384 \end{bmatrix}$$

$$\|\boldsymbol{\Delta}(\mathbf{s})\|_\infty = \|\mathbf{G_R}(\mathbf{s}) - \mathbf{G}(\mathbf{s})\|_\infty \leq \mathbf{2}(\sigma_\mathbf{3} + \sigma_\mathbf{4}) = \mathbf{1.8594}$$

Model validation

Production rate

Nonlinear Reduced-order Full-order



Input disturbances: zero mean, 15% of nominal values

State-shared model

Let

$$\mathbf{F_j} = \begin{bmatrix} 0 & 1 \\ \\ -a_1 & -a_2 \end{bmatrix} \quad \mathbf{G_j} = \begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} \qquad \mathbf{j = 1, 2, 3}$$

with $a_1 = a_2 = 1$ such that the pair (F, G) are controllable.

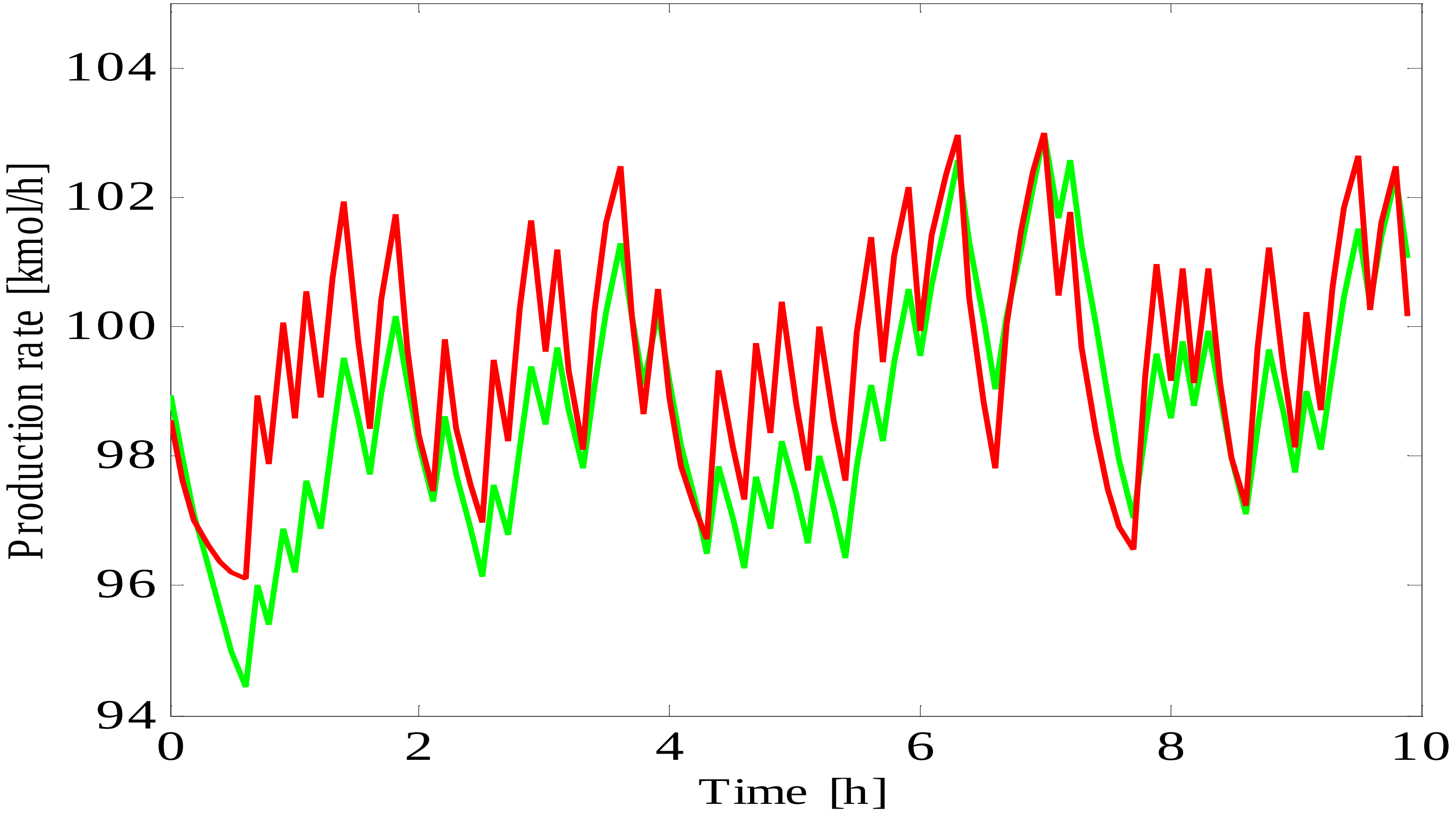
$$\mathbf{F} = \begin{bmatrix} F_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & F_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & F_3 \end{bmatrix}_{\mathbf{6\times 6}} \qquad \mathbf{G} = \begin{bmatrix} G_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G_3 \end{bmatrix}_{\mathbf{6\times 3}}$$

Adaptable parameters: $\mathbf{n(m^2 + 1) = 20}$

Model validation

Production rate

State-shared Nonlinear



Input disturbances: zero mean, 15% of nominal values

Average relative error (ARE)

$$\text{ARE}(j) = \frac{\sum_{i=1}^{k} \left| \frac{y_p(i,j) - \bar{y}_p(j) - y(i,j)}{y_p(i,j)} \right|}{k}$$

$y_p(i, j)$ is the j^{th} output of the nonlinear system at sample point i.

% ARE

	P	y_{A3}	F_4
NL-FL	0.02	0.01	0.07
FL-RL	0.01	0.30	0.83
NL-RL	0.02	0.29	0.93
SS-RL	0.01	0.04	0.18
NL-SS	0.25	0.32	0.93

NL: nonlinear system, FL: full-order linear model, RL: reduced-order linear model, SS: state-shared model

Transition control: SSMPC

- M = 4, P = 10
- Wy = [(P, 3), (yA3, 1), (F4, 10)]
- Wu = 1
- Output constraints:

— 2400 < P < 2950 kPa

— $0 < y_{A3} < 100$ %

— $0 < F_4 < 200$ kmol/hr

- Input constraints:

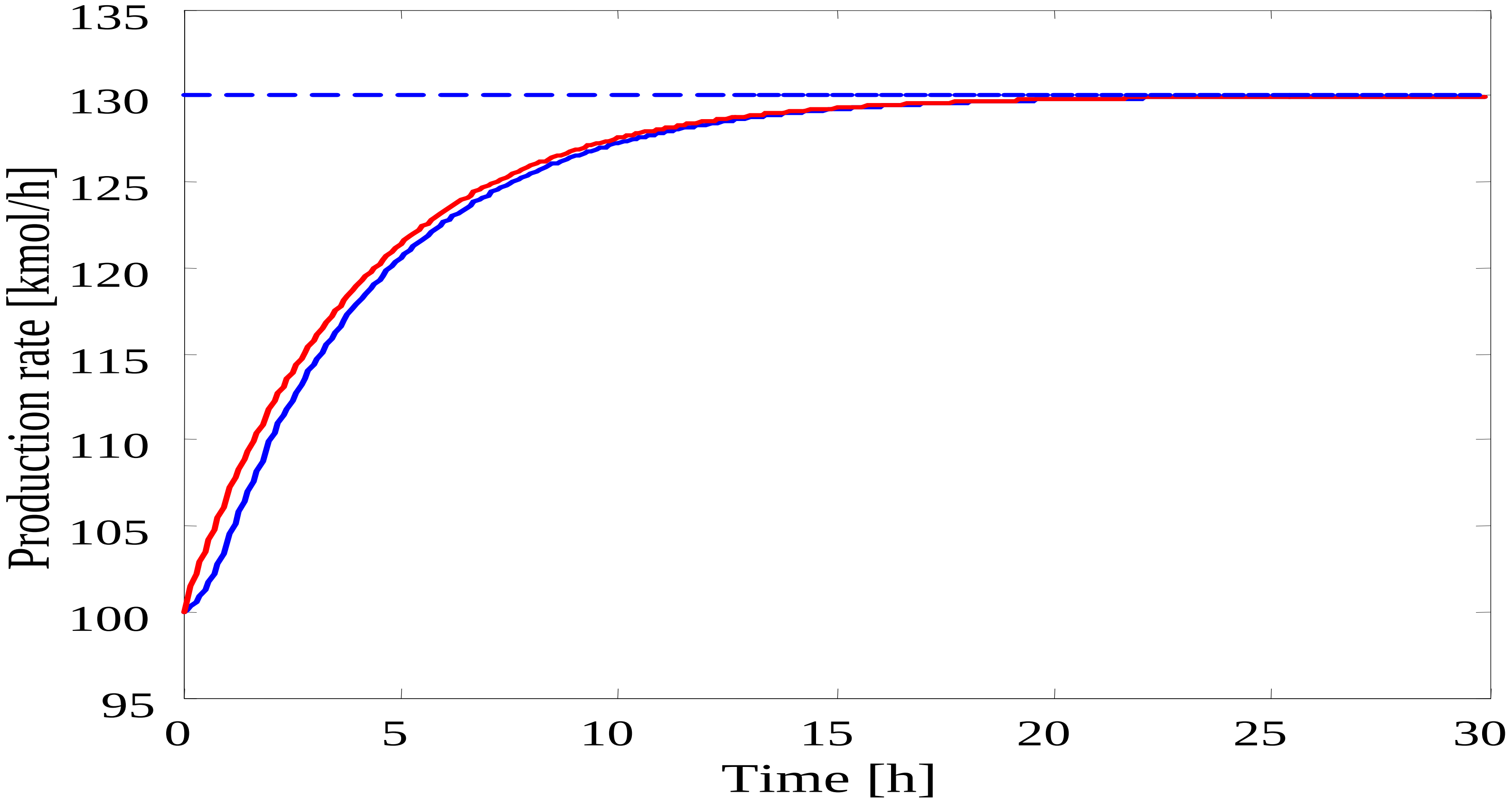
— $-50 < \Delta\mathrm{F}_1 < 80$ kmol/hr

— $-5 < \Delta\mathrm{F}_2 < 10$ kmol/hr

— $-7 < \Delta\mathrm{F}_3 < 10$ kmol/hr

Transition: OPI → OPII

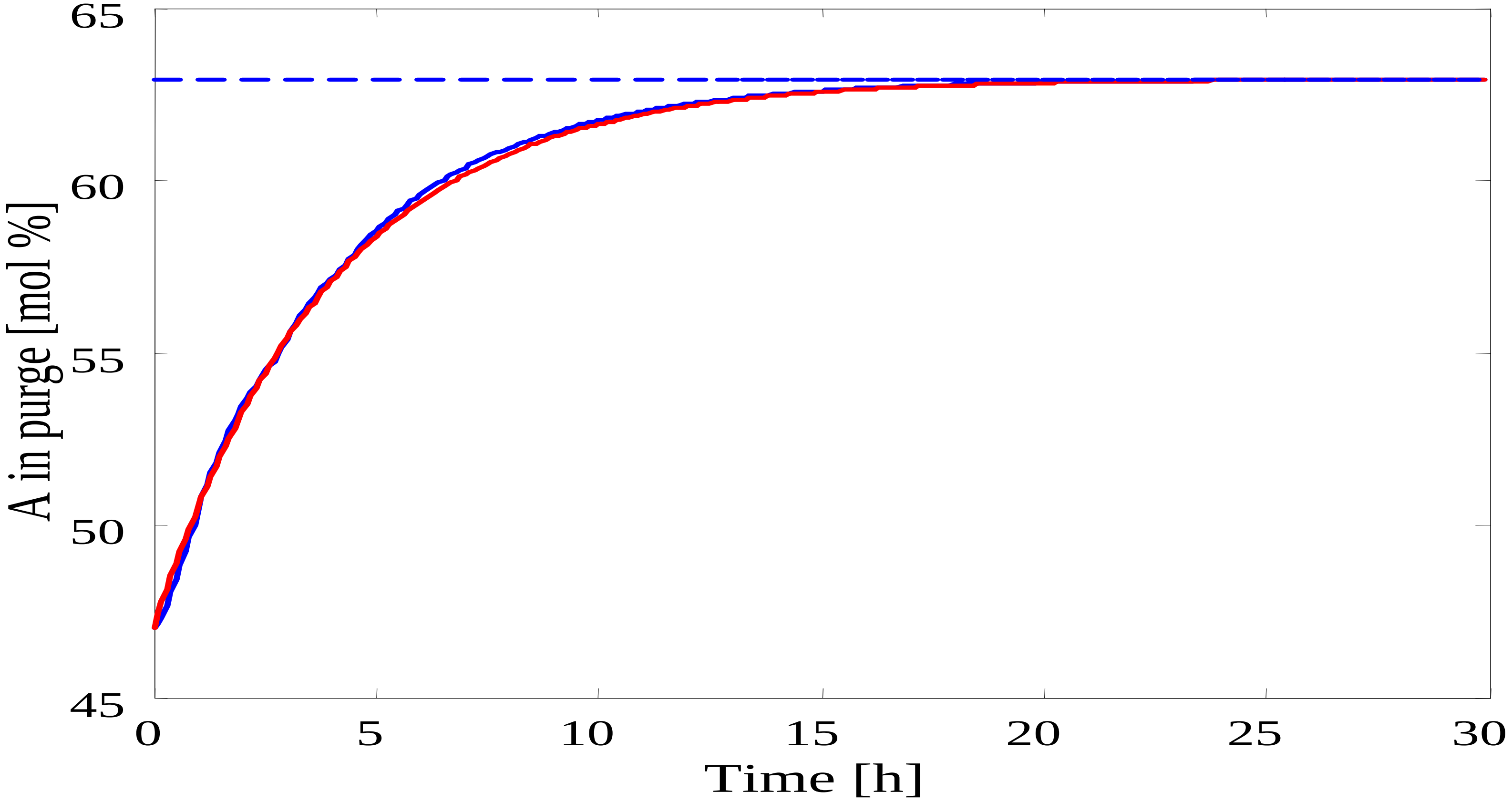
Production rate, kmol/h



reference
nonlinear system
Dashed line: setpoint

Transition: OPI → OPII

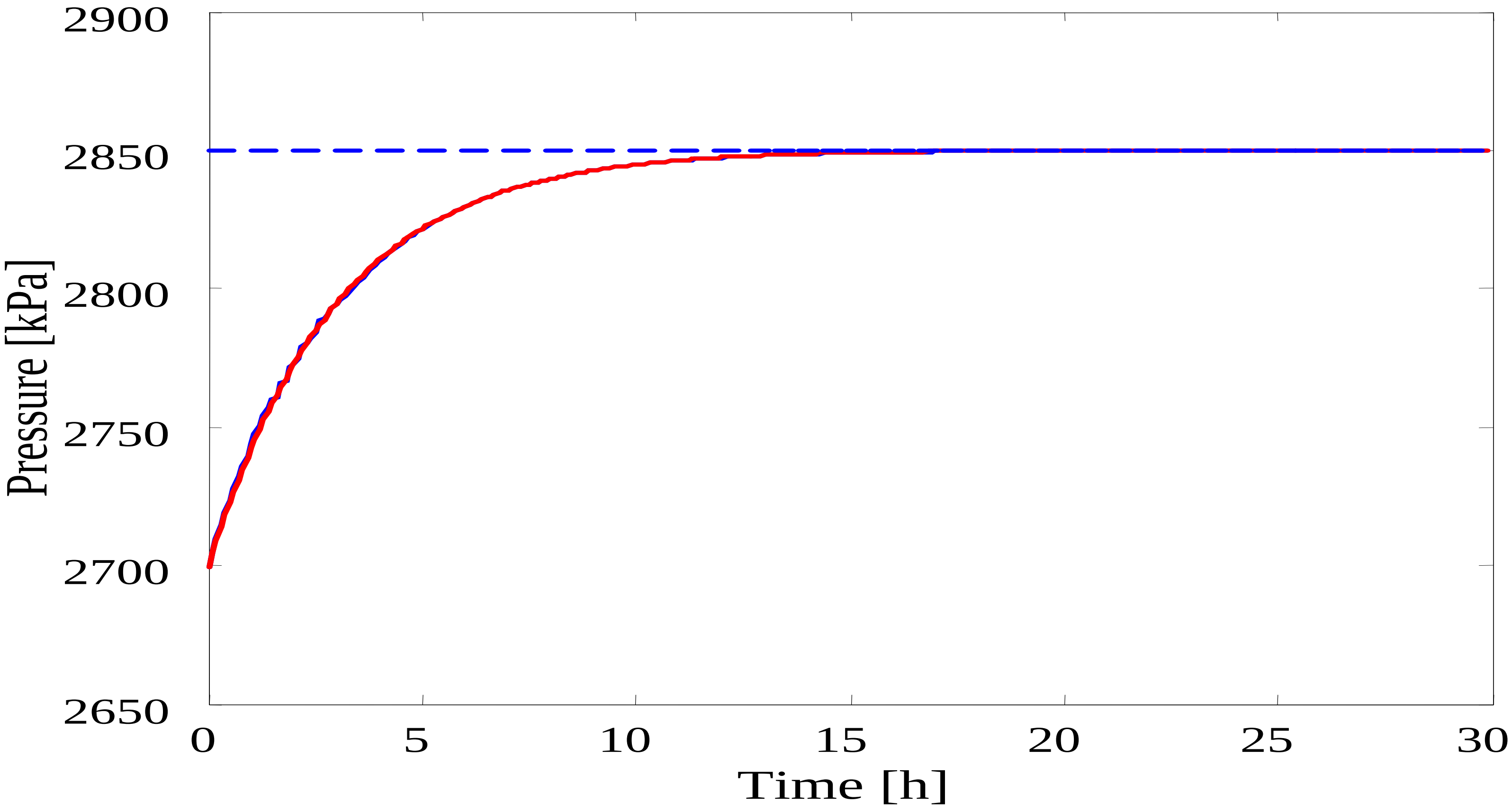
Component A in purge, mol%



reference
nonlinear system
Dashed line: setpoint

Transition: OPI → OPII

Pressure, kPa



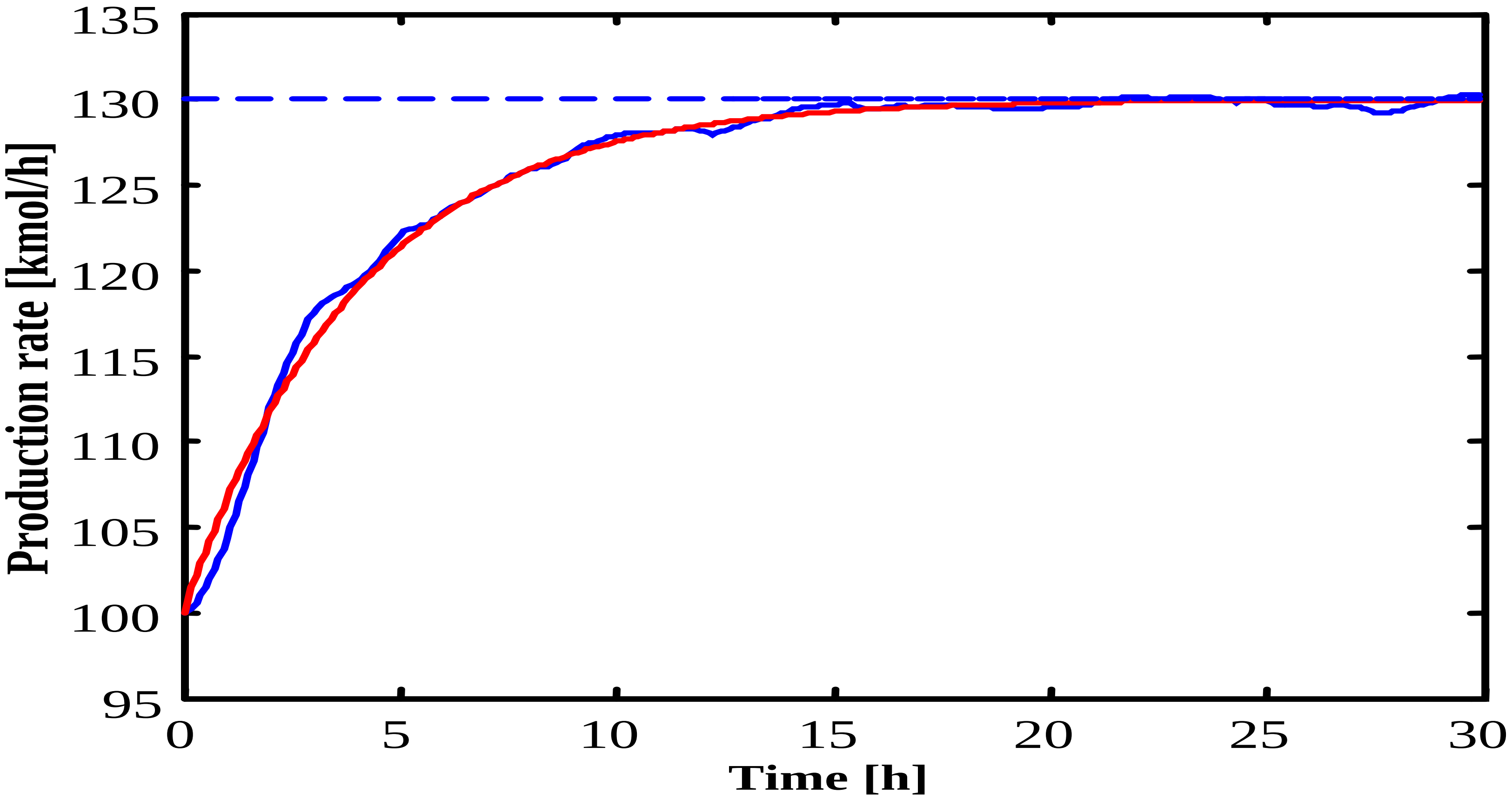
reference
nonlinear system
dashed line: setpoint

Transition: Unmodeled output disturbances

- M = 4, P = 10
- Wy = [(P, 3), (yA3, 1), (F4, 10)]
- Wu = 1
- Output and input constraints
- Unmeasured output disturbances with SNR=10:1
- First order filter

Transition: OPI → OPII

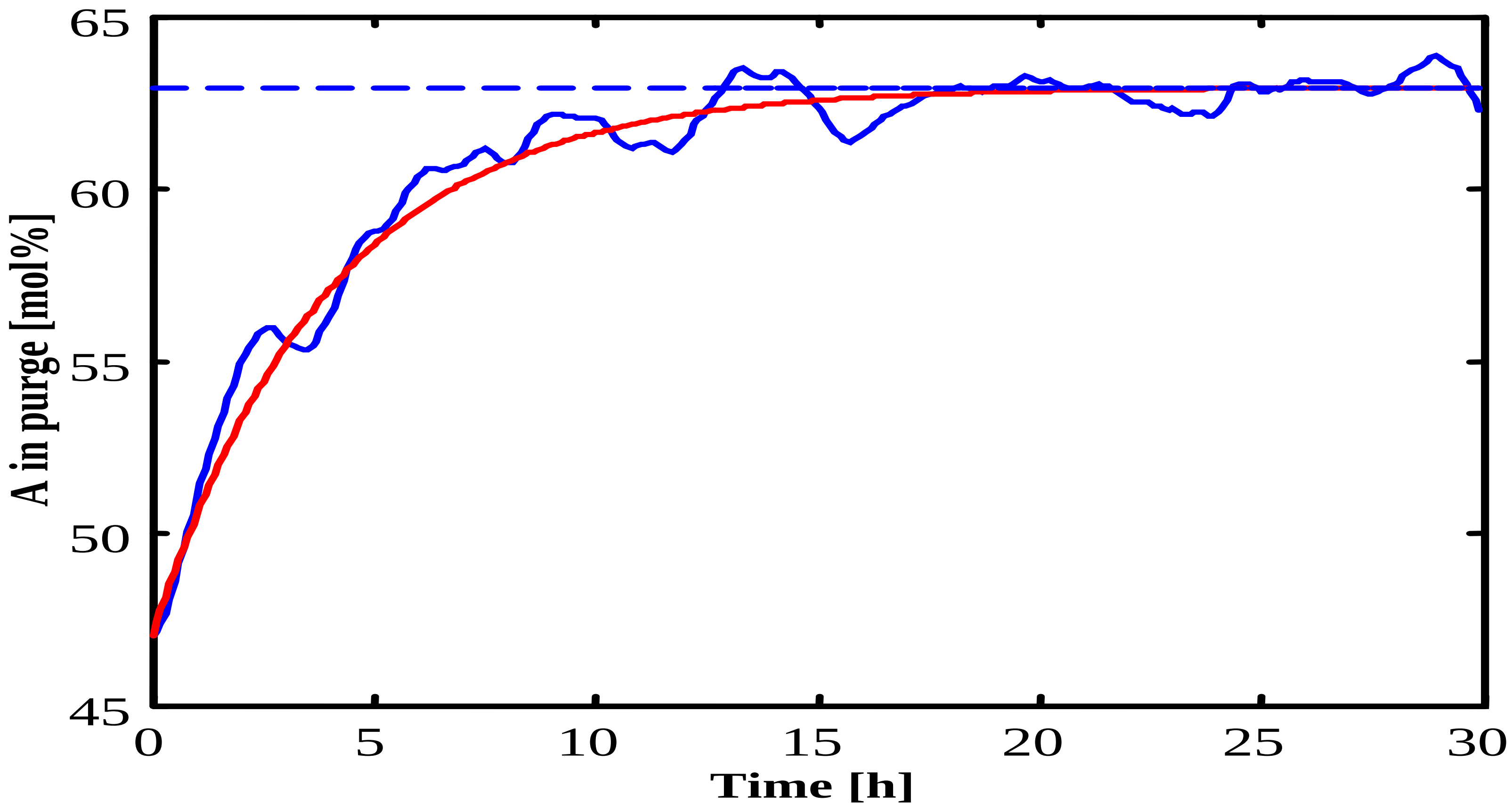
Production rate, kmol/h



reference
nonlinear system
Dashed line: setpoint

Transition: OPI → OPII

Component A in purge, mol%



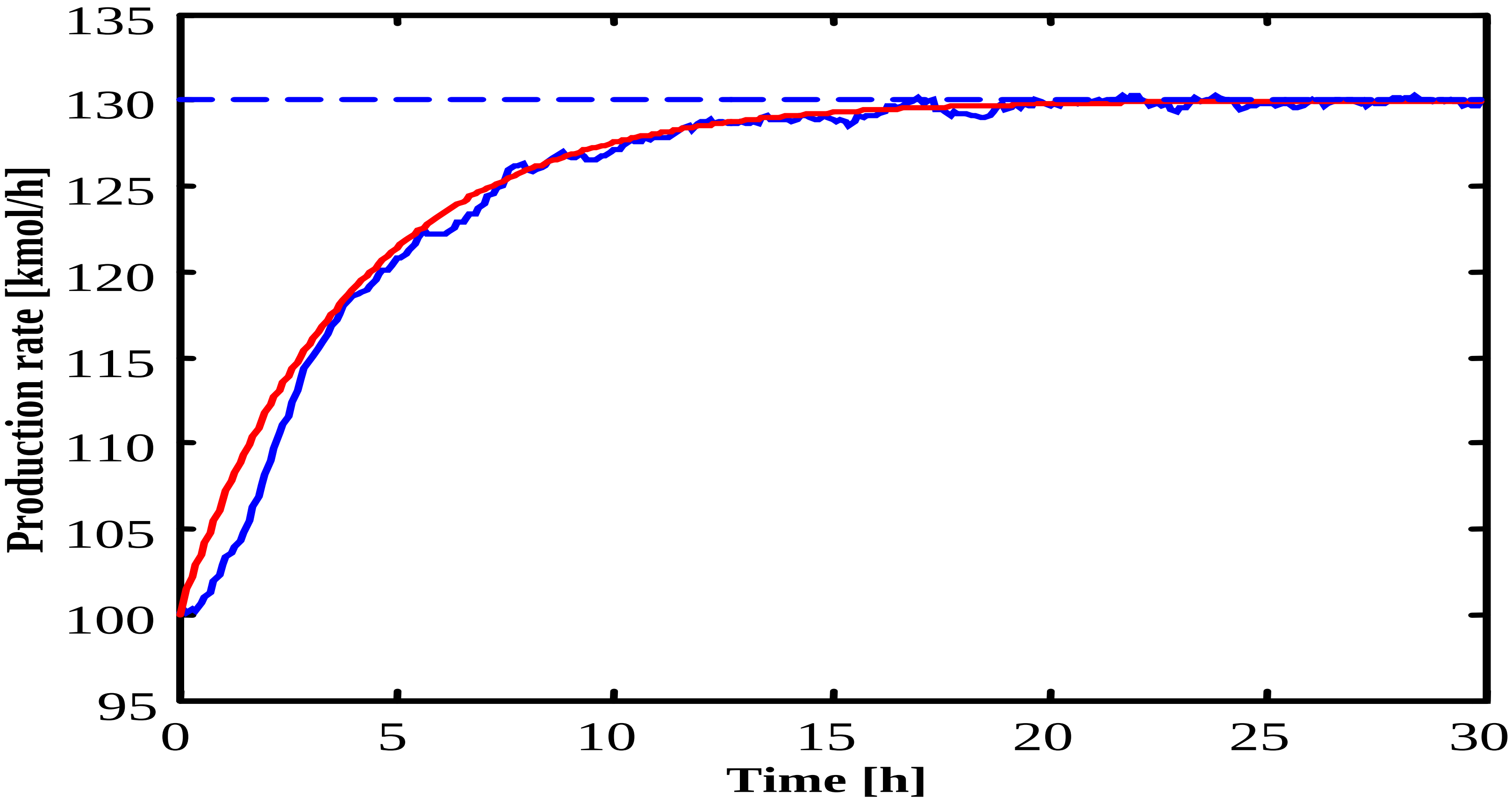
reference
nonlinear system response
Dashed line: setpoint

Transition: Comp. A in the purge is not measured

- M = 4, P = 10
- Wy = [(P, 3), (yA3, 1), (F4, 10)]
- Wu = 1
- Output and input constraints
- Unmeasured output disturbances
- First order filter
- Component A in the purge, y_{A3}, is estimated from input/output data.

Transition: OPI → OPII

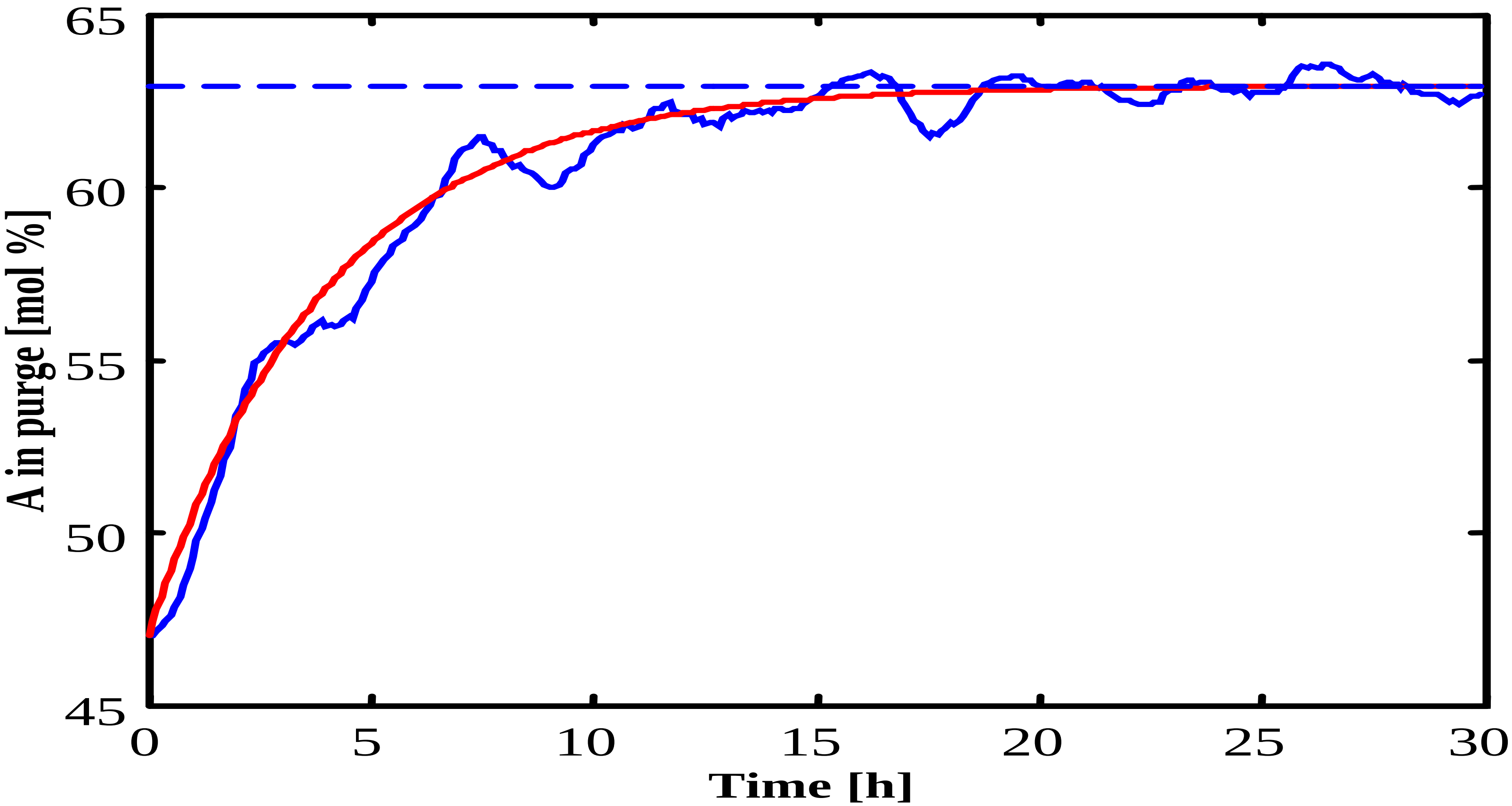
Production rate, kmol/h



reference
nonlinear system
Dashed line: setpoint

Transition: OPI → OPII

Component A in purge, mol%



reference
nonlinear system
Dashed line: setpoint

Summary

- Development of a compact representation of a system that has many operating states: State Shared Model
  - Represent known and unknown operating states
  - Each operating state has a different measurement equation
  - Adapt the parameters of the measurement equation $n(m^2+1)$
- Transition Control (SSMPC)
  Production change MIMO nonlinear reactor system
  - Unmeasured disturbances
  - Unmeasured output

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